Designs, strategies and cliques

Pierre Boudes, IML, Marseille

Tokyo, march 2004

Introduction

- Interaction (social life of proofs)

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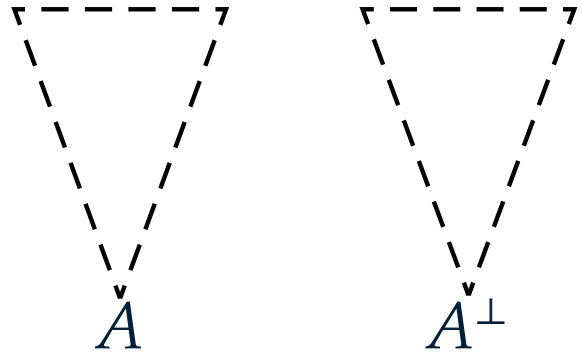

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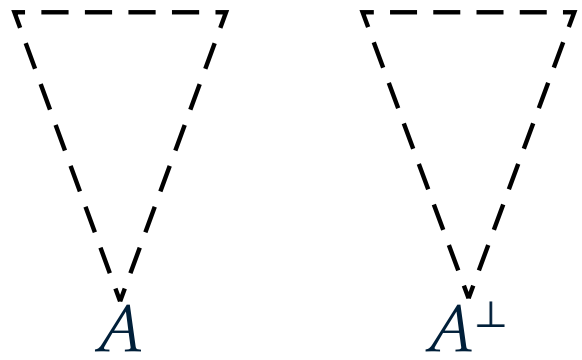

- Polarisation

Introduction

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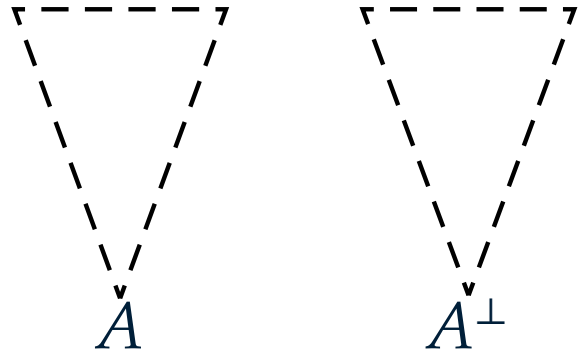
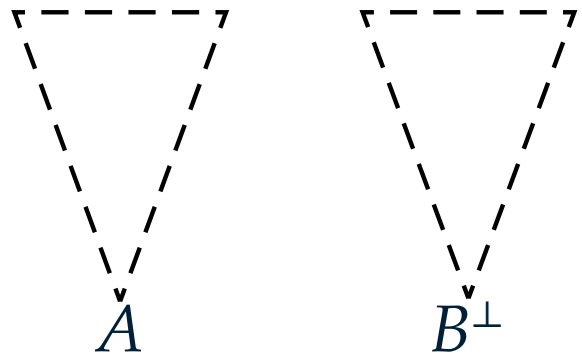


- Polarisation
- Localization

Introduction

- Interaction (social life of proofs)



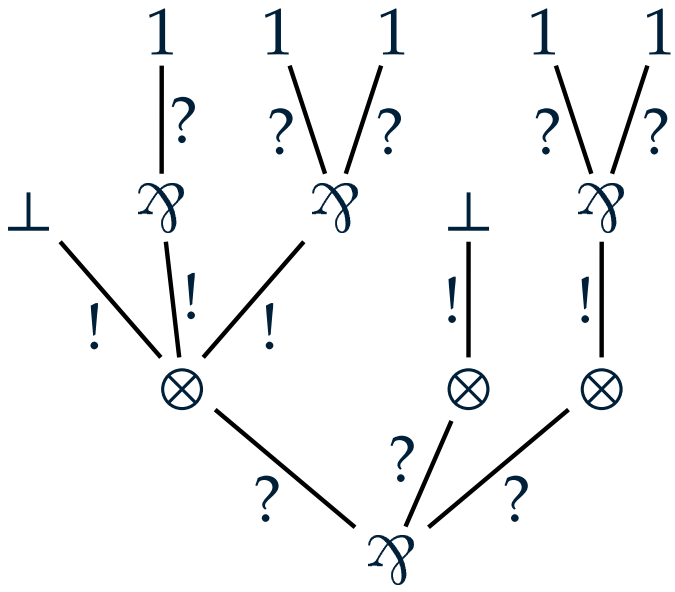
- Polarisation
- Localization

Scope

- Polarized
- No atoms
- No additives ($\&, \oplus, \top, 0$)
- No data copying (*ie* LL contraction rule) so no real exponentials

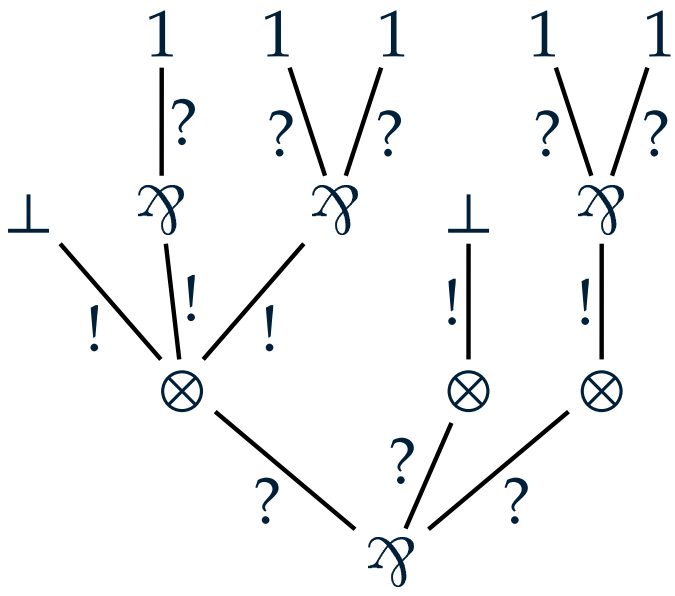
Ludics : formulae

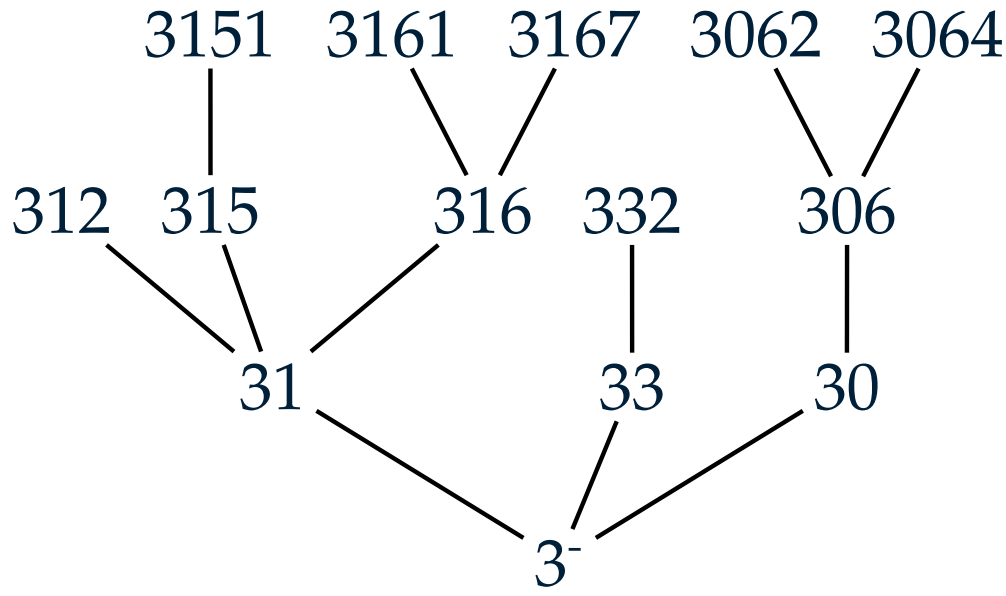
$$F = ?(!\bot \otimes !?1 \otimes !(?1 \parr ?1)) \parr ?!\bot \parr ?!(?1 \parr ?1)$$



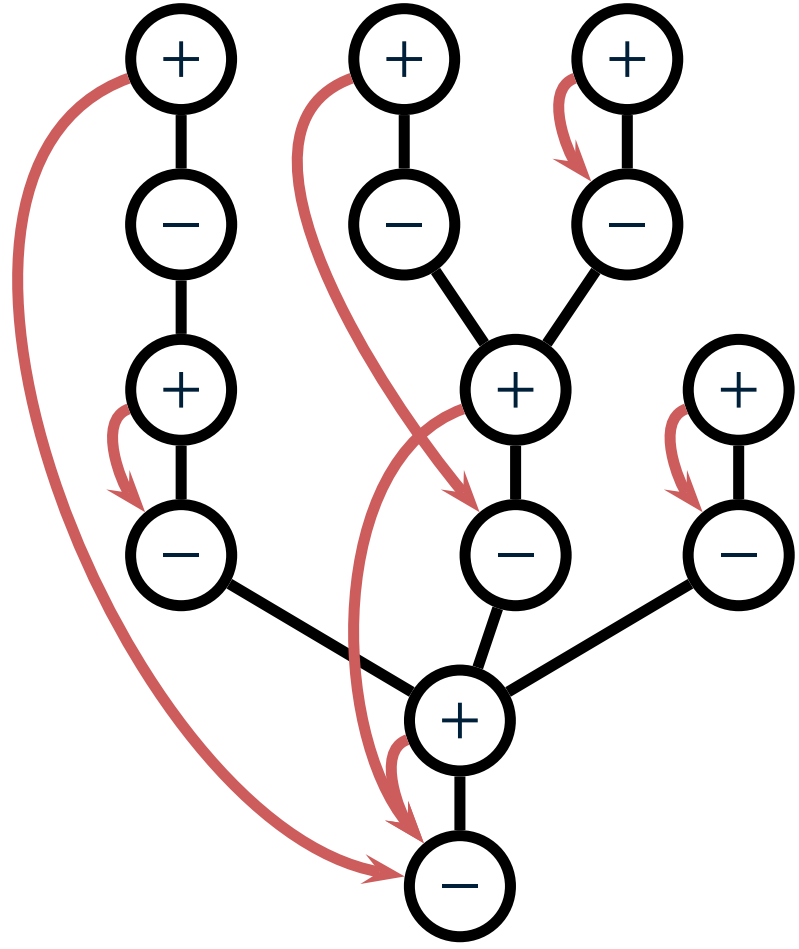
Ludics : formulae

$$F = ?(!\bot \otimes !?1 \otimes !(?1 \parr ?1)) \parr ?!\bot \parr ?!(?1 \parr ?1)$$



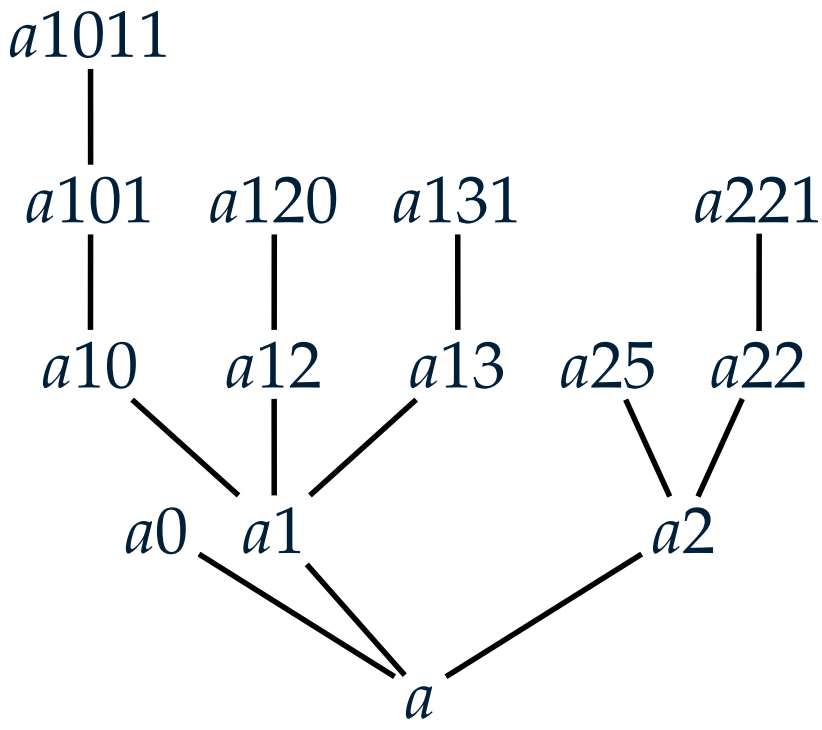


Designs, abstract Böhm trees, ...

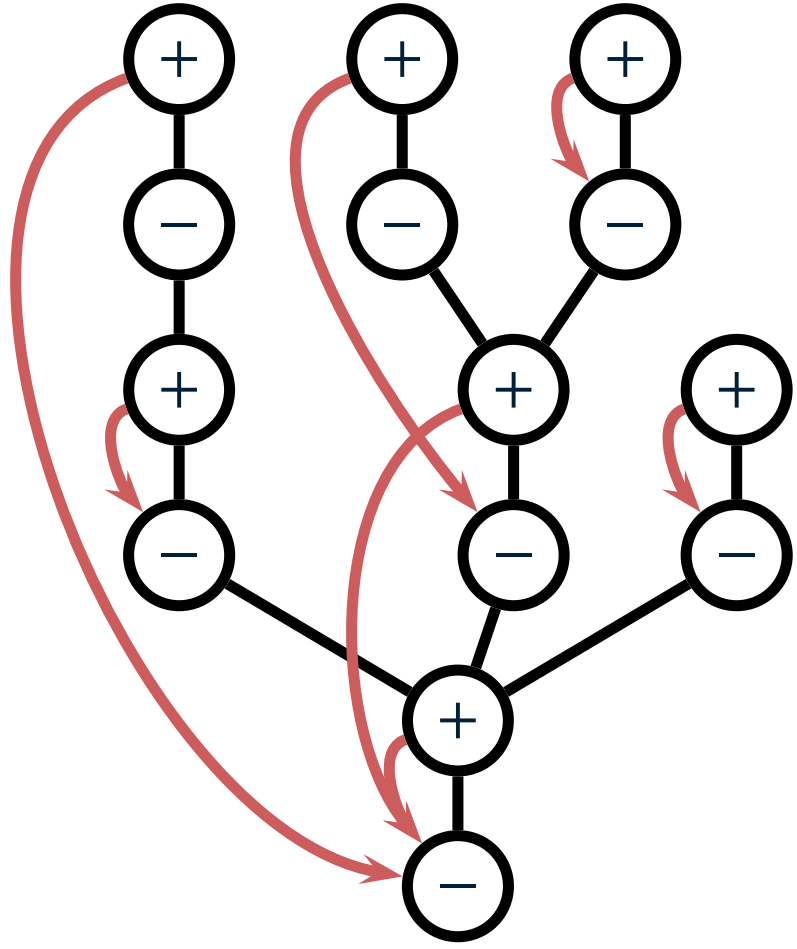


Proof

Designs, abstract Böhm trees, ...



Formula



Proof

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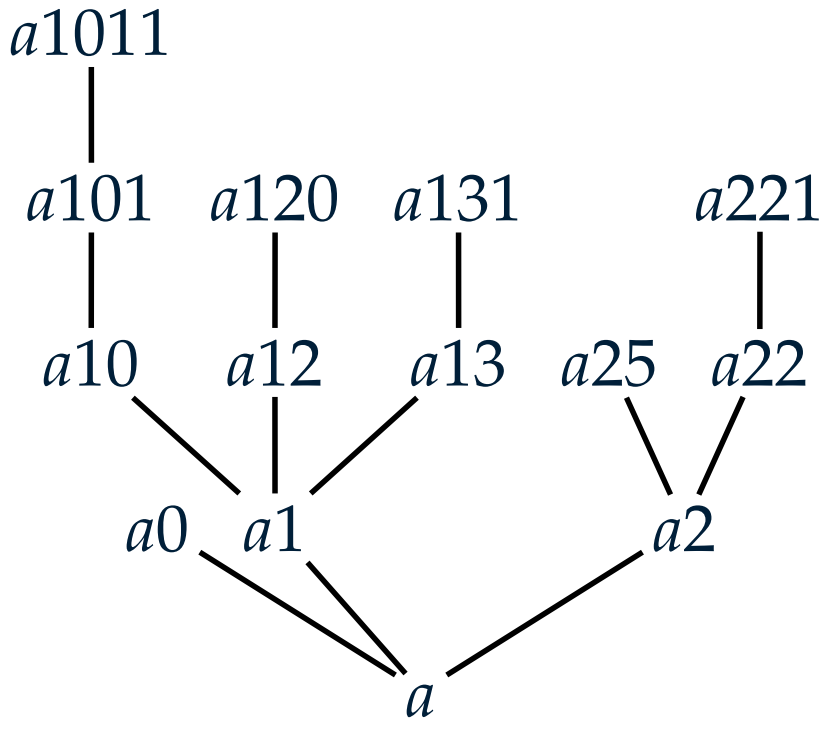
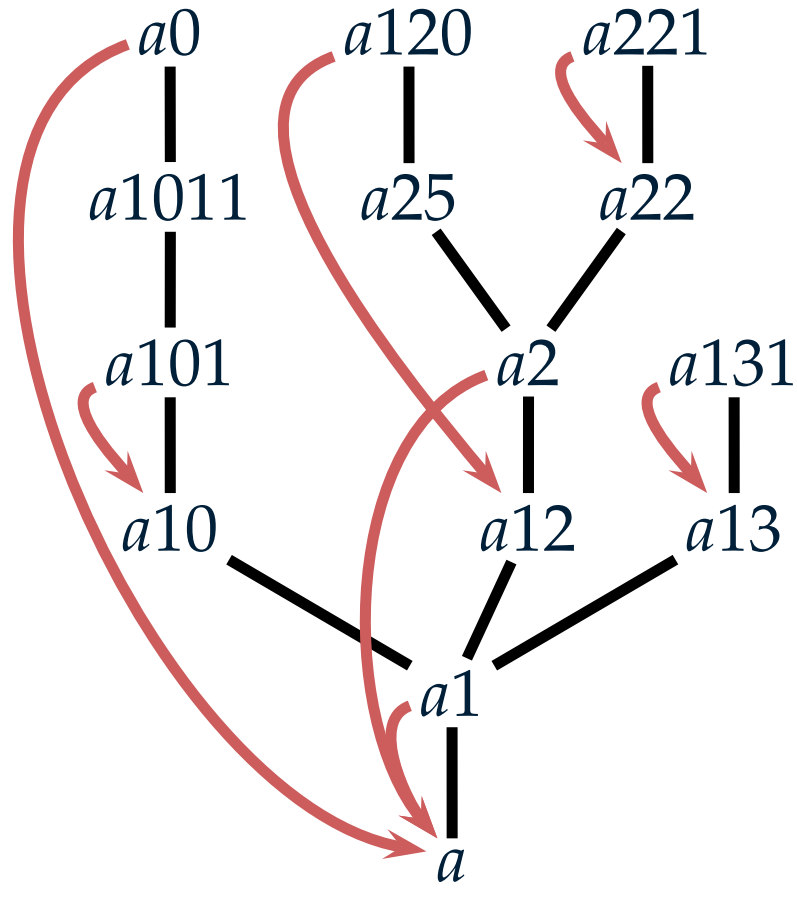


Formula



Proof

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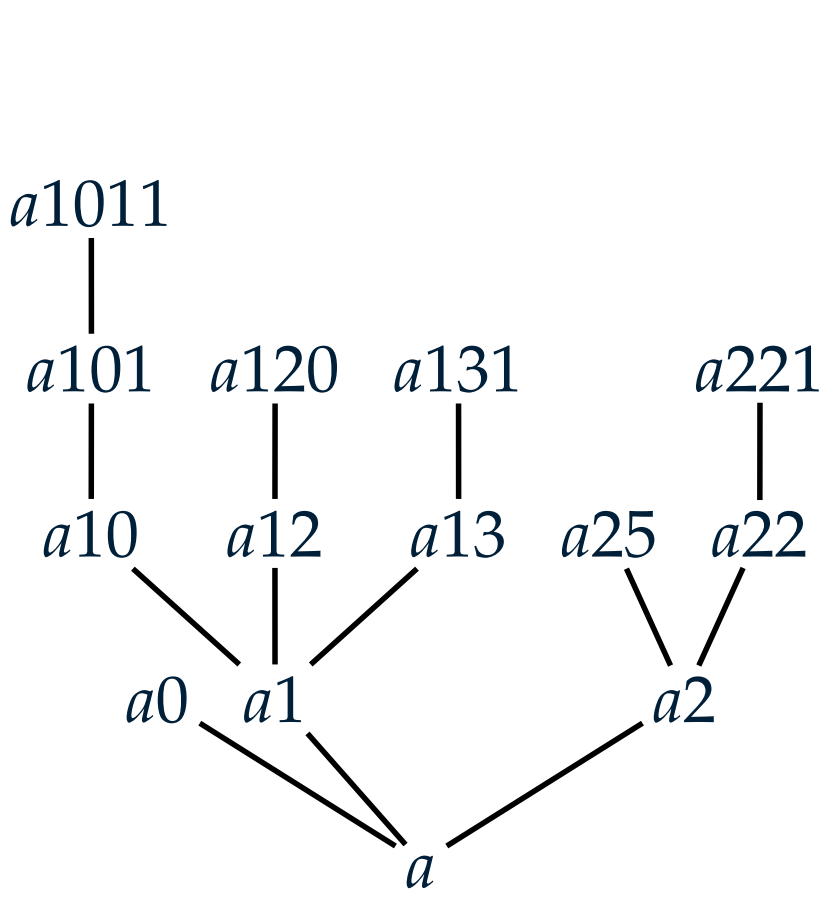
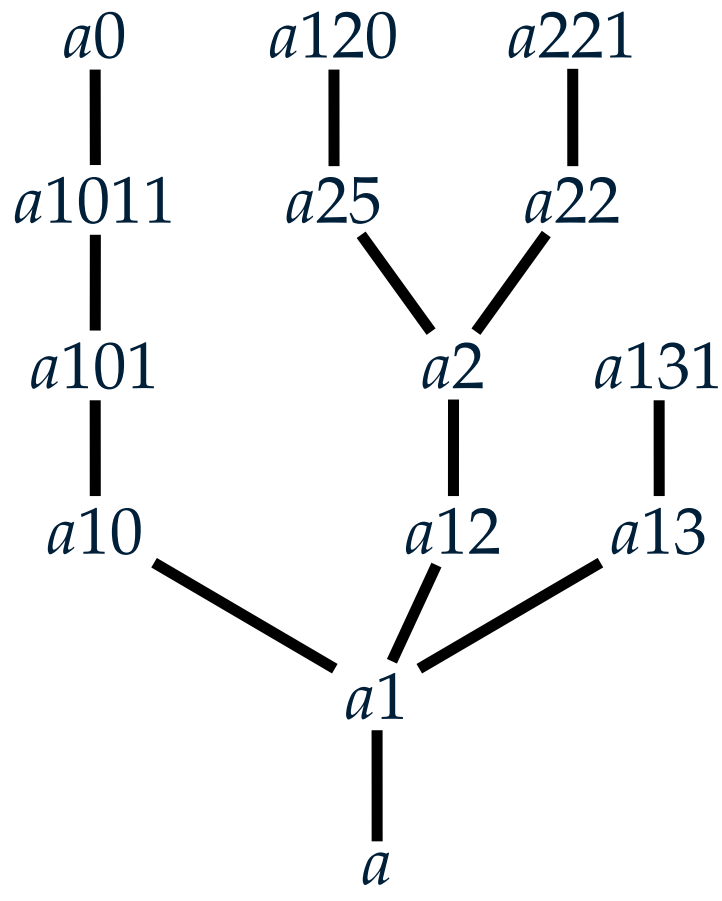


Formula



Proof

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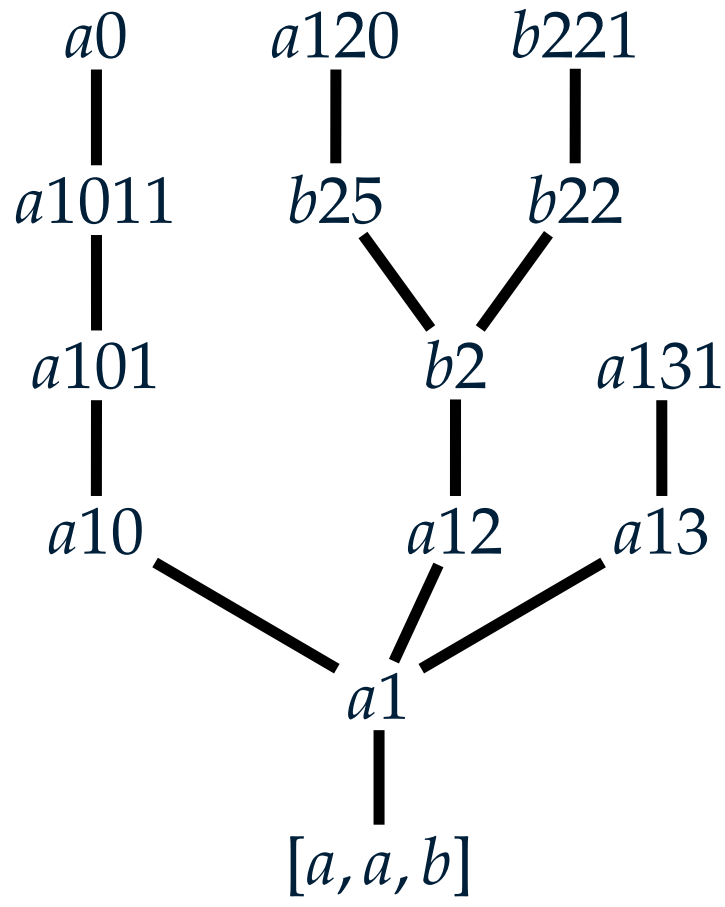
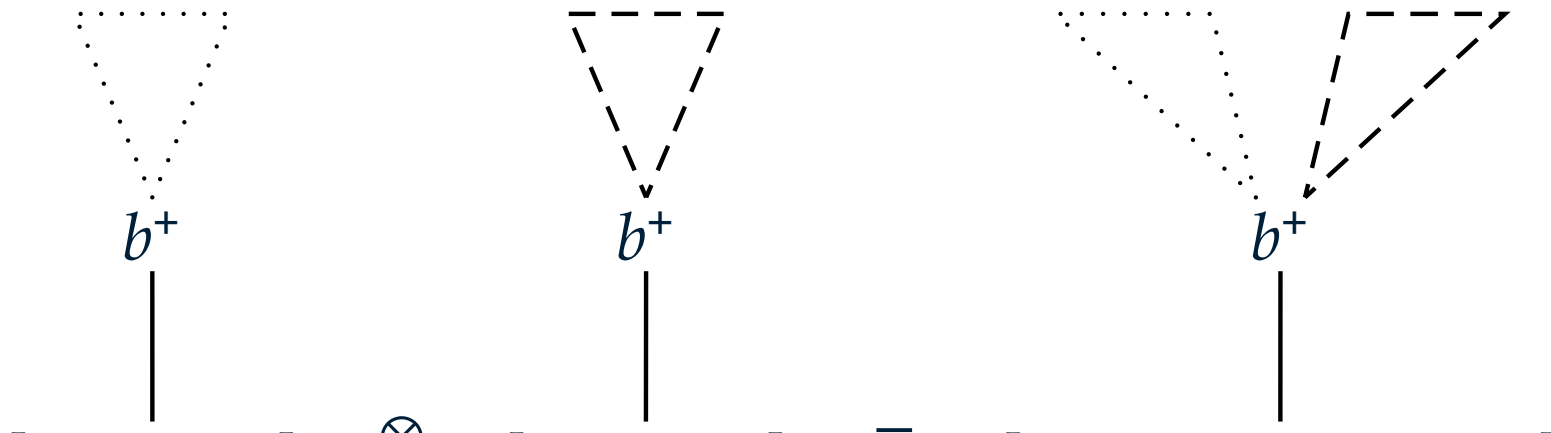


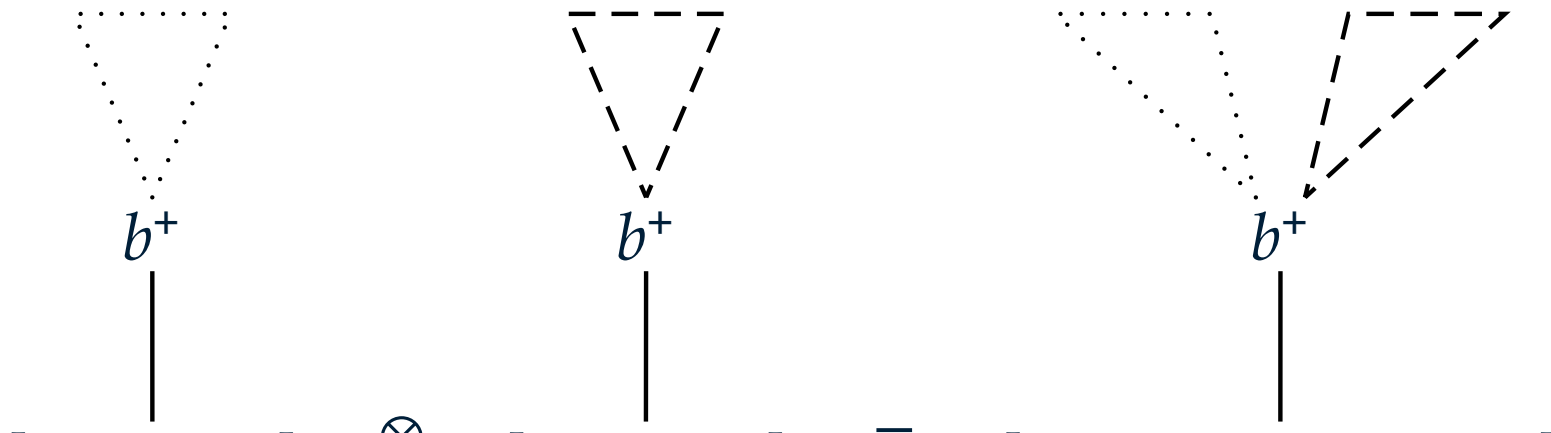
Proof

Tensor



$$\frac{\vdash \Gamma, P(b) \qquad \vdash \Delta, P'(b)}{\vdash \Gamma, \Delta, P \otimes P'(b)}$$

Tensor

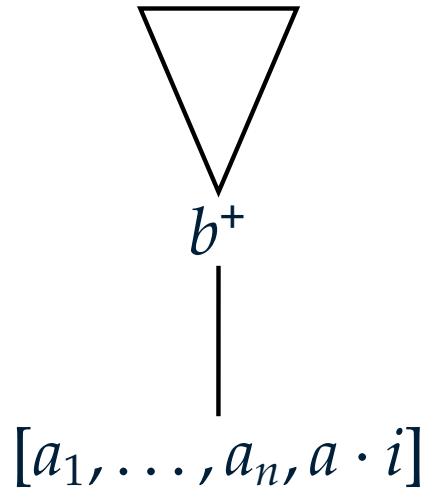


$$\frac{\vdash \Gamma, P(b) \qquad \vdash \Delta, P'(b)}{\vdash \Gamma, \Delta, P \otimes P'(b)}$$

$$\frac{}{\vdash 1(a)}$$

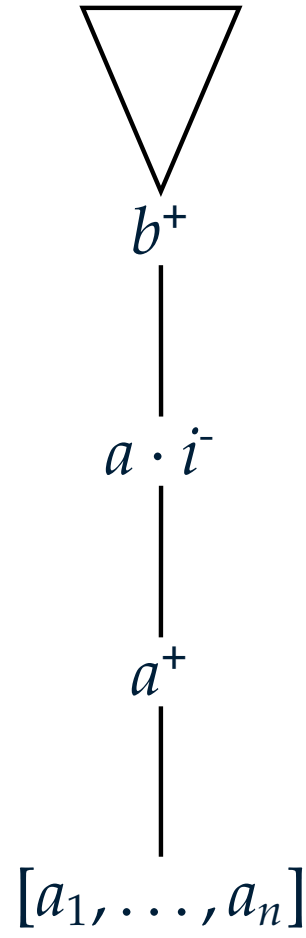
(0-ary case)

Of course

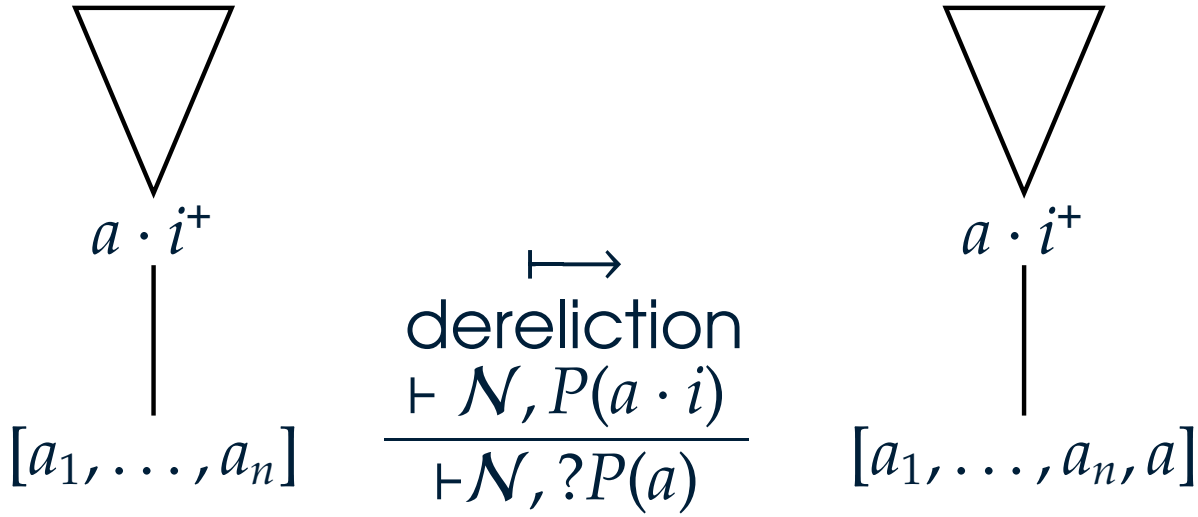


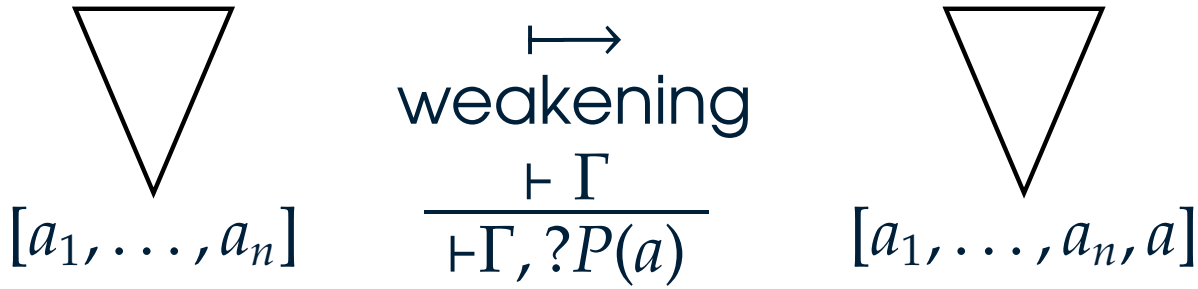
$$\longmapsto$$

$$\frac{\vdash \mathcal{N}, N(a \cdot i)}{\vdash \mathcal{N}, !N(a)}\ \text{promotion}$$

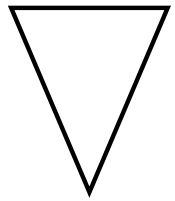


Dereliction, weakening





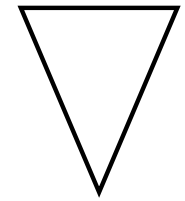
Par



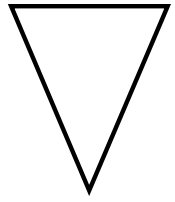
$[a_1, \ldots, a_n, a, a]$

$$\longmapsto$$

$$\text{Par}\ \frac{\vdash \Gamma, N(a), N'(a)}{\vdash \Gamma, N \parr N'(a)}$$



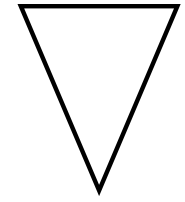
$[a_1, \ldots, a_n, a]$



$[a_1, \ldots, a_n]$

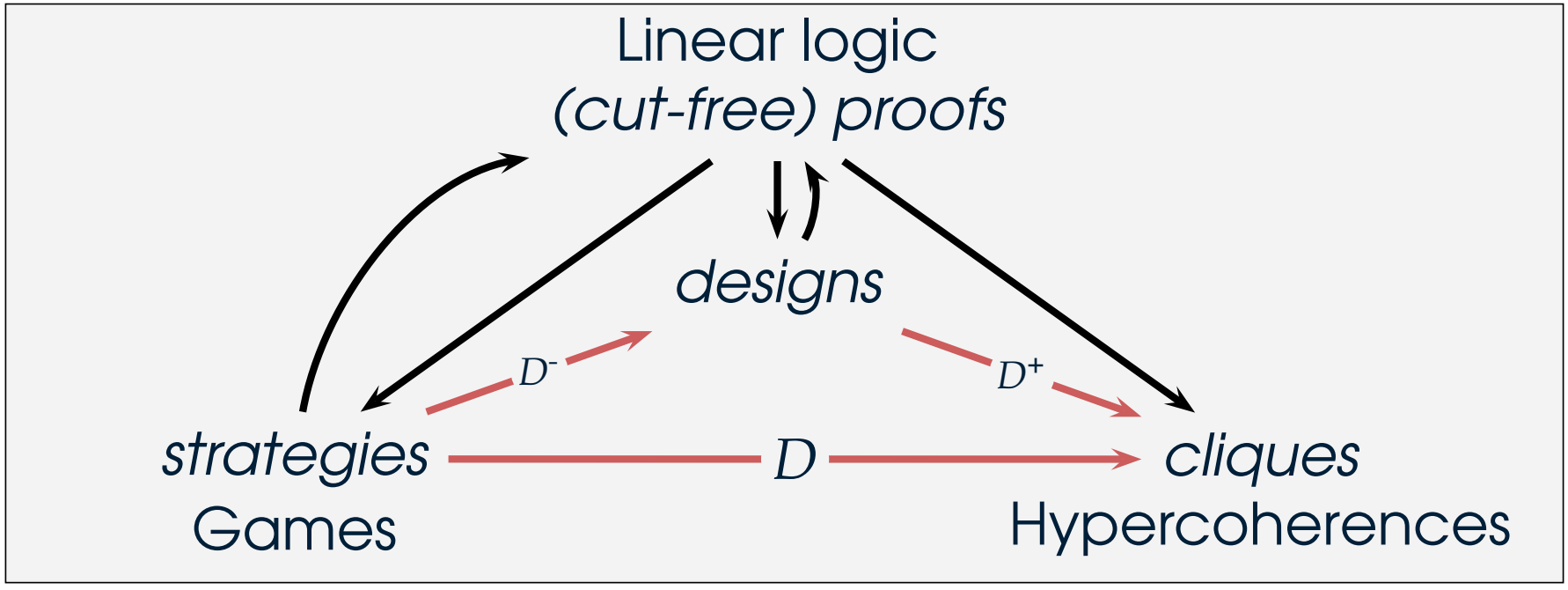
$$\longmapsto$$

$$\text{bot}\ \frac{\vdash \Gamma}{\vdash \Gamma, \bot(a)}$$

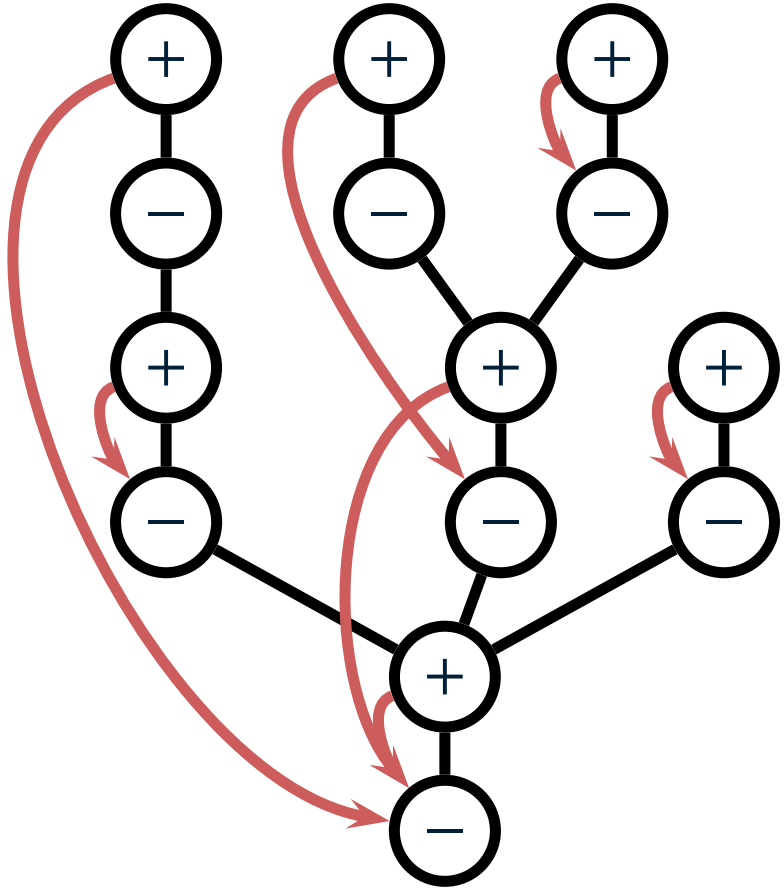


$[a_1, \ldots, a_n, a]$

Situation


Linear logic
(cut-free) proofs
designs
D^-
D^+
strategies
D
cliques
Games
Hypercoherences

Relation with HO games (Laurent's polarized games)



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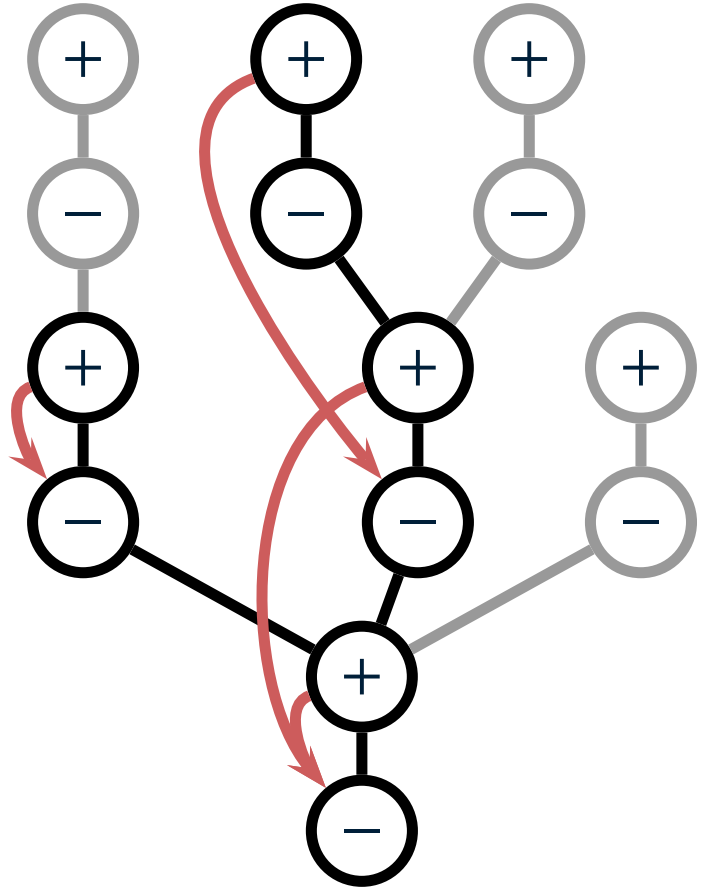

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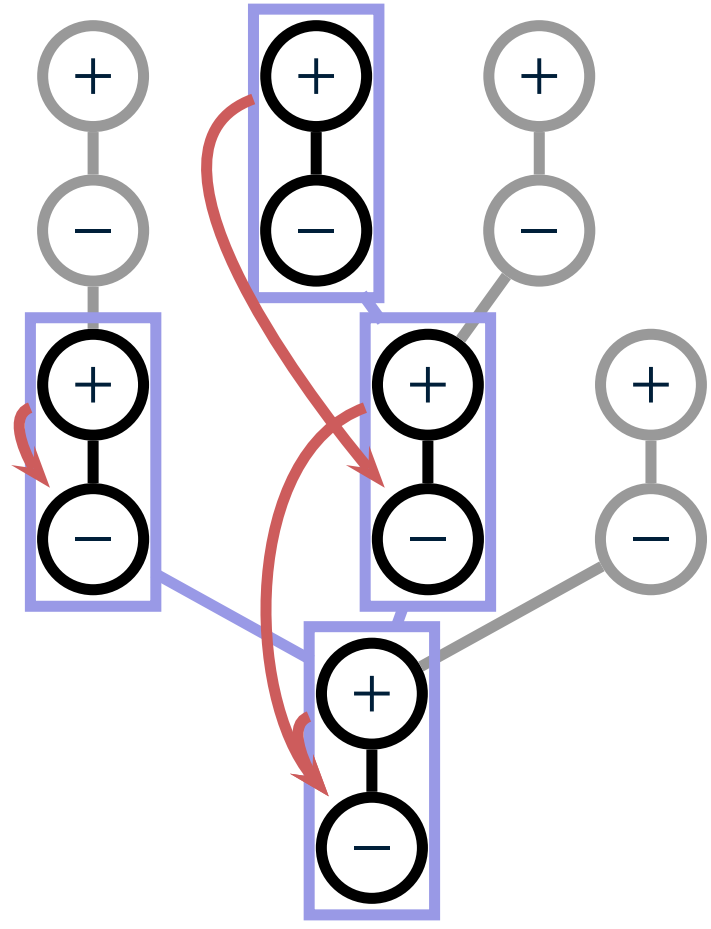

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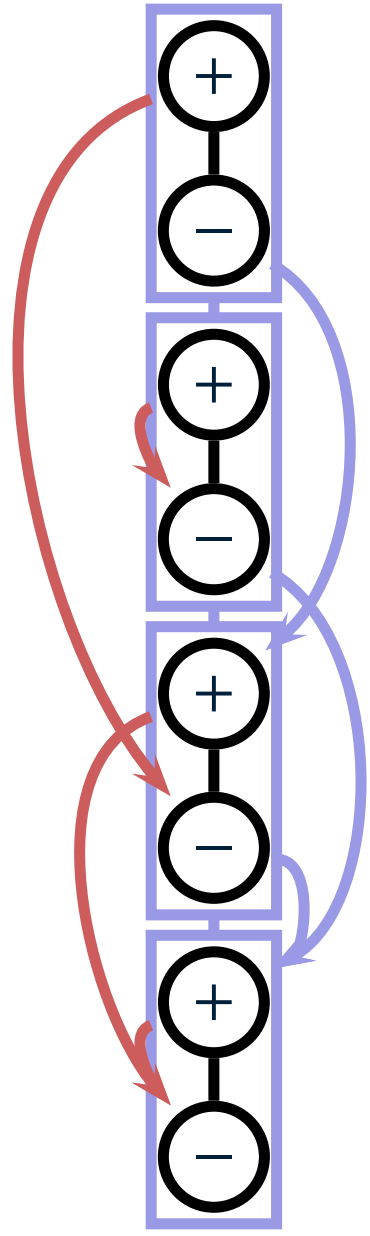
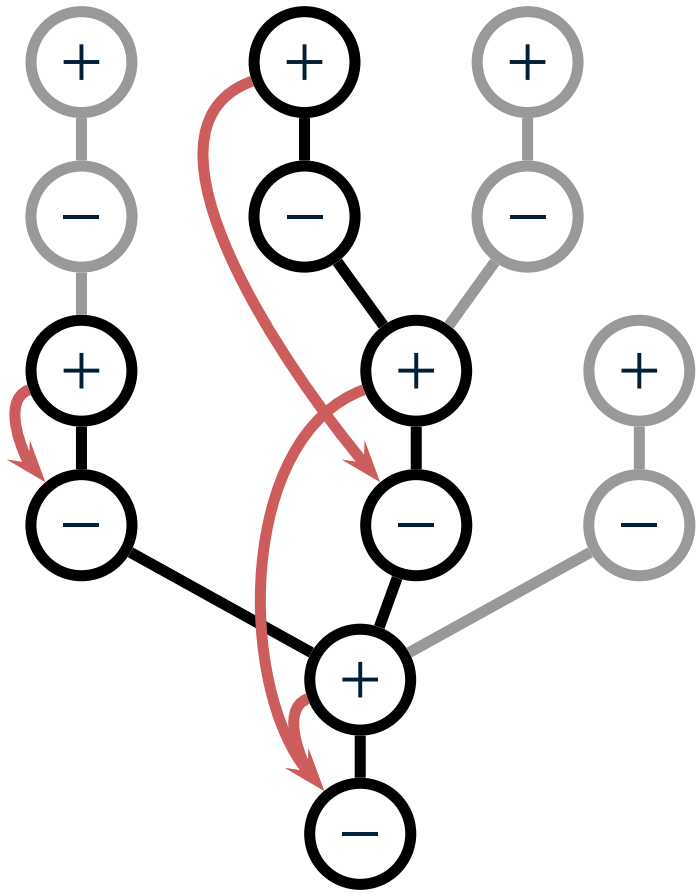


Relation with the relational model



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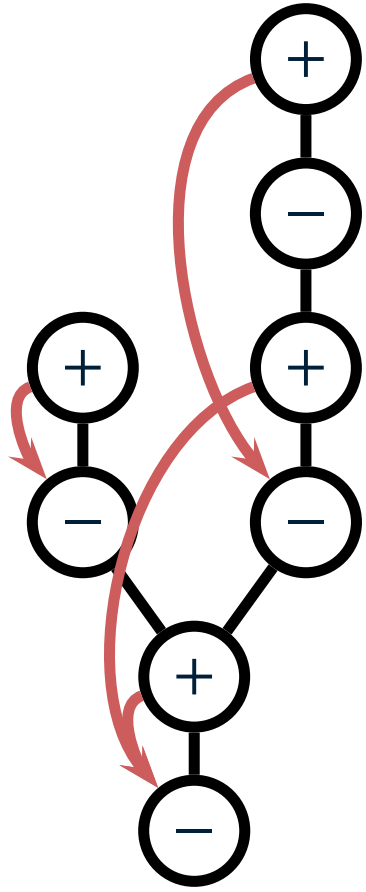

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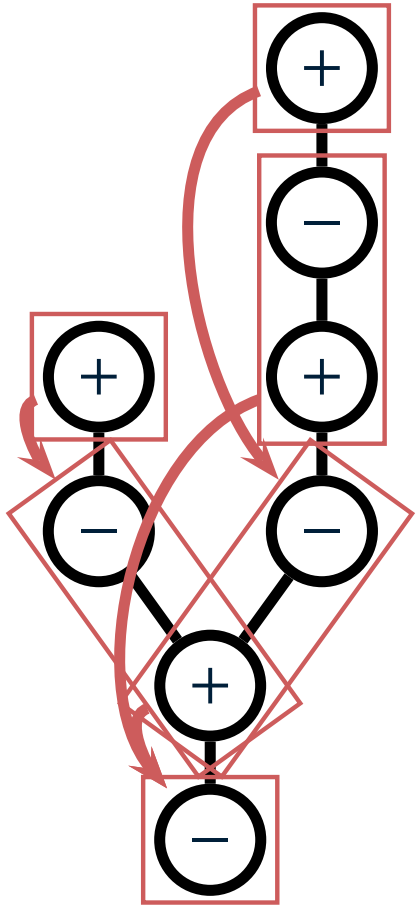

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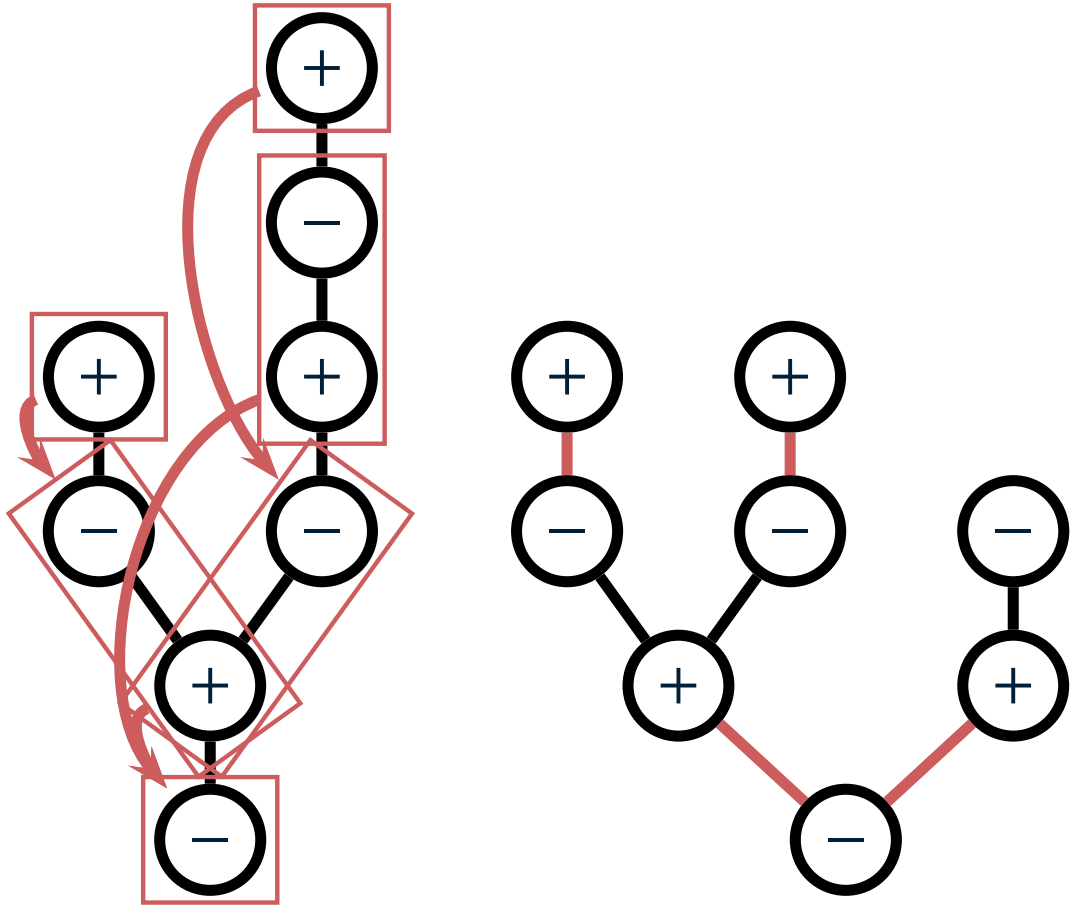
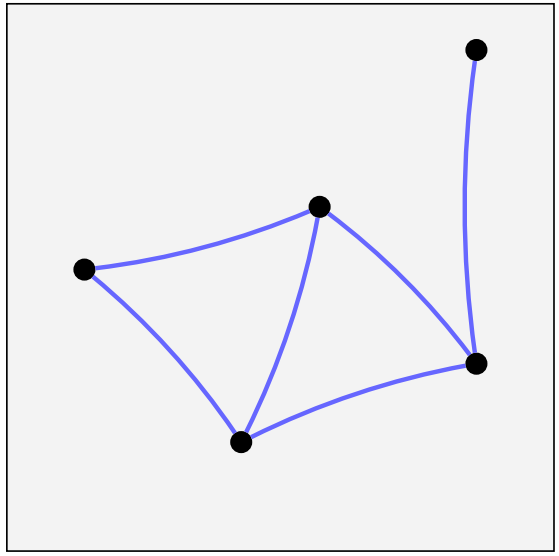
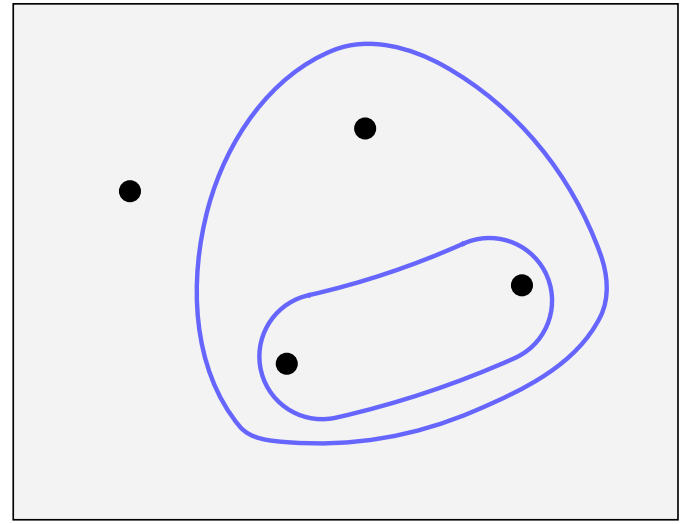


Coherence and hypercoherence spaces

a coherence

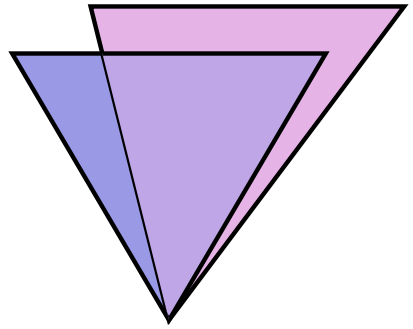


a hypercoherence



Coherence

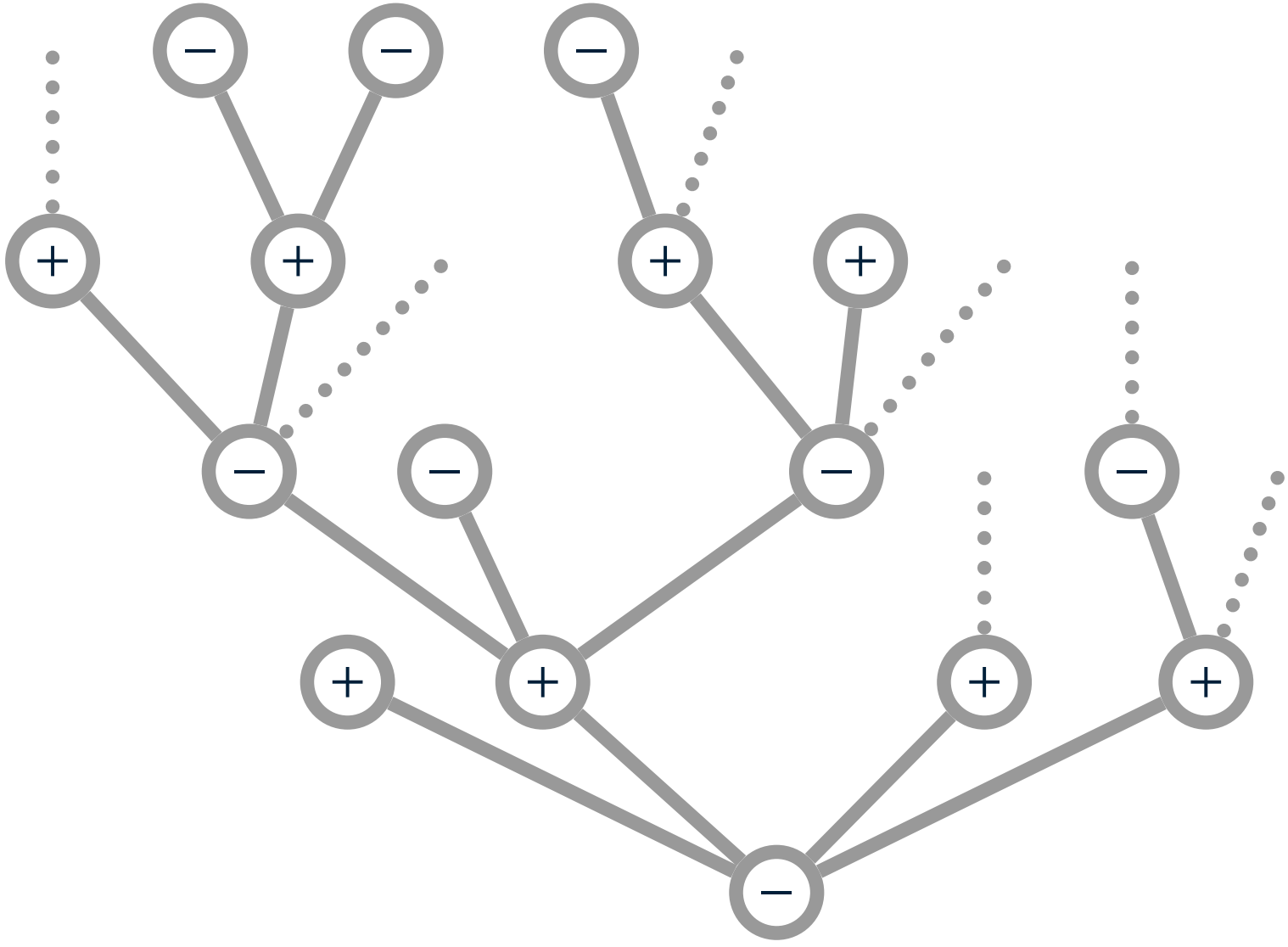
- Coherence of $p_0, \ldots, p_n$ depends only on $\cap p_i$ (intersection) and $\cup p_i$ (surperposition) and is equivalent to the coherence of $\cap p_i$ and $\cup p_i$.



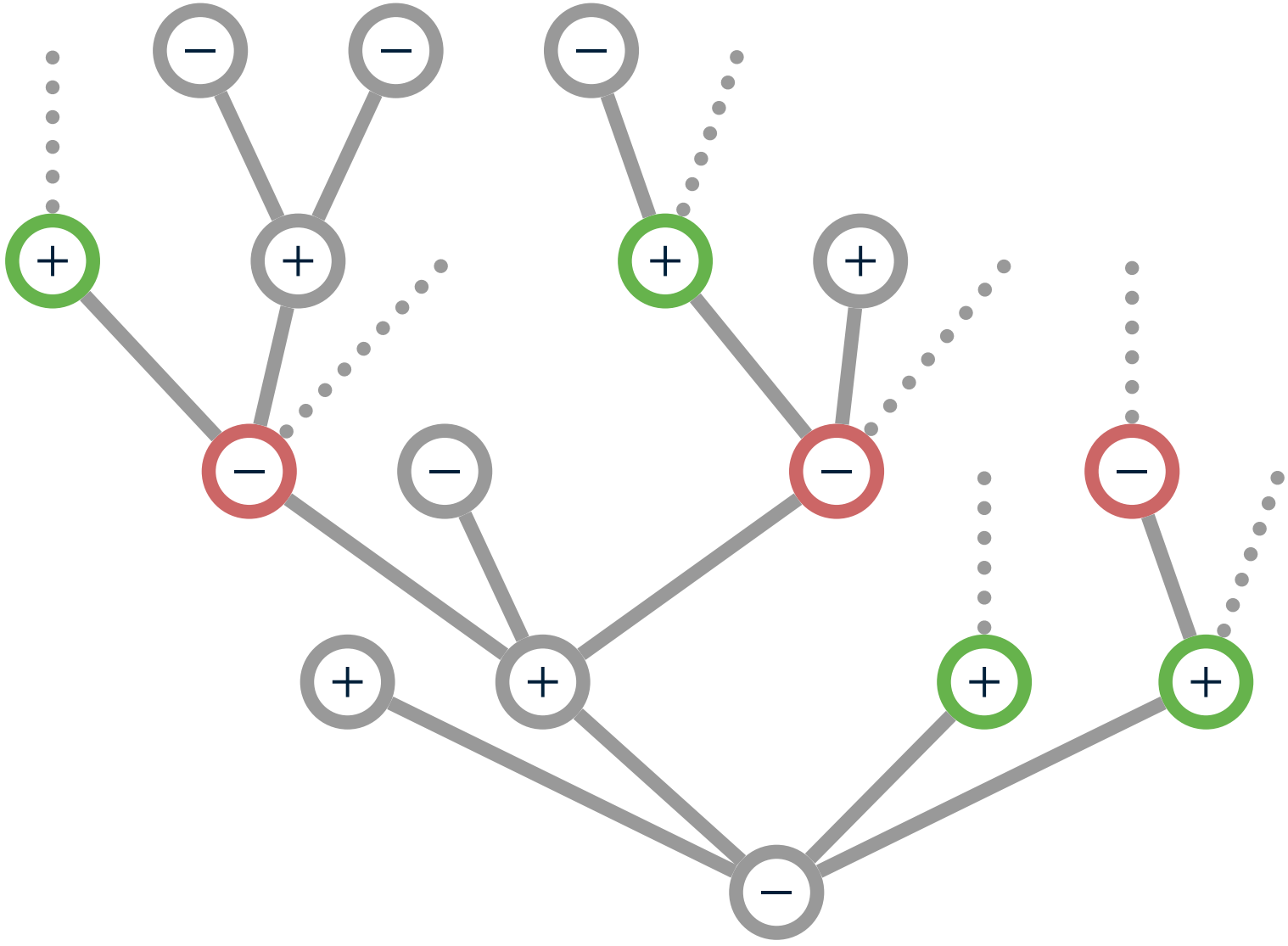
- Interpretations of proofs are closed by intersection and superposition of trees/points.

(. . .)

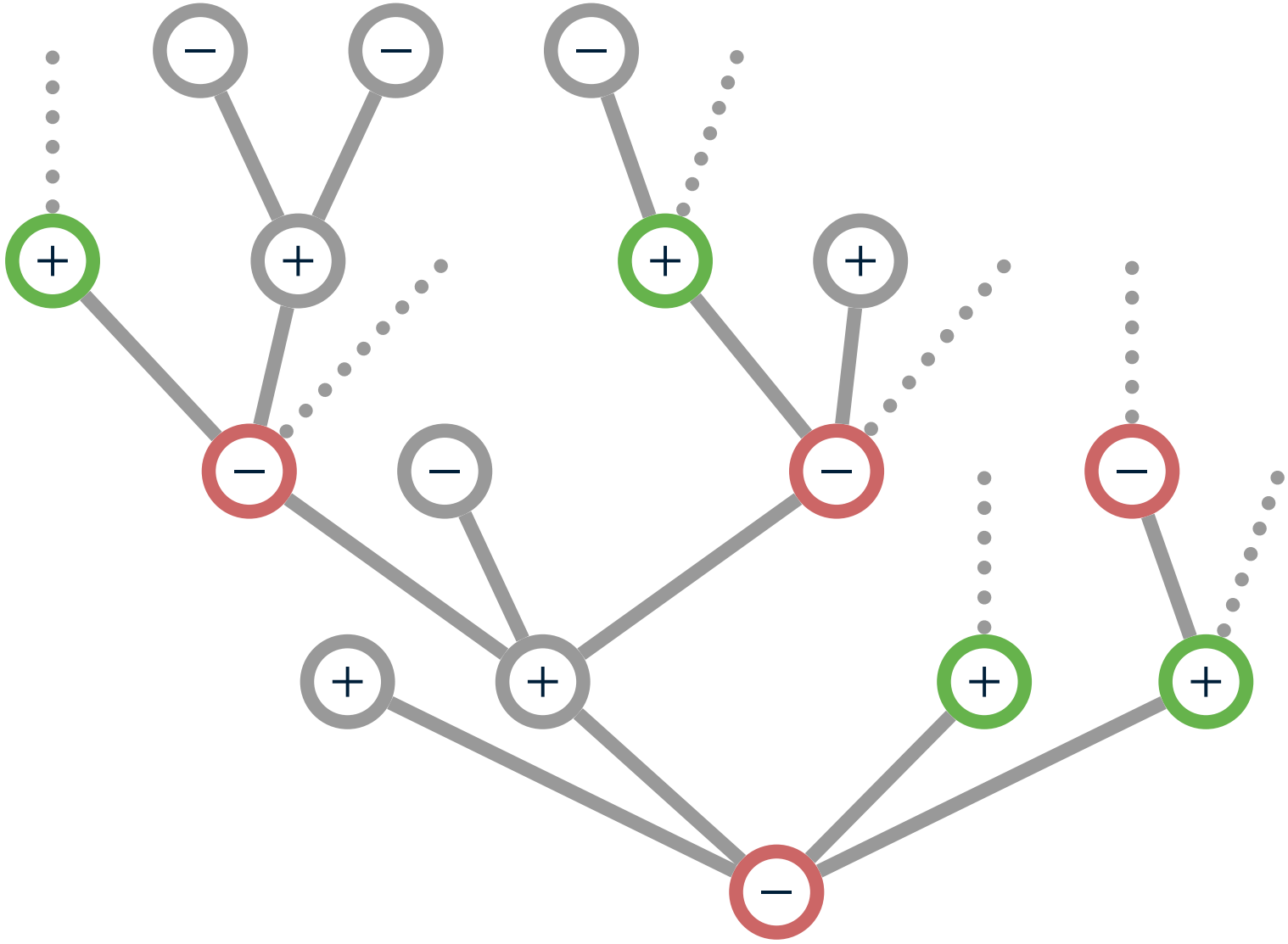
Coherence



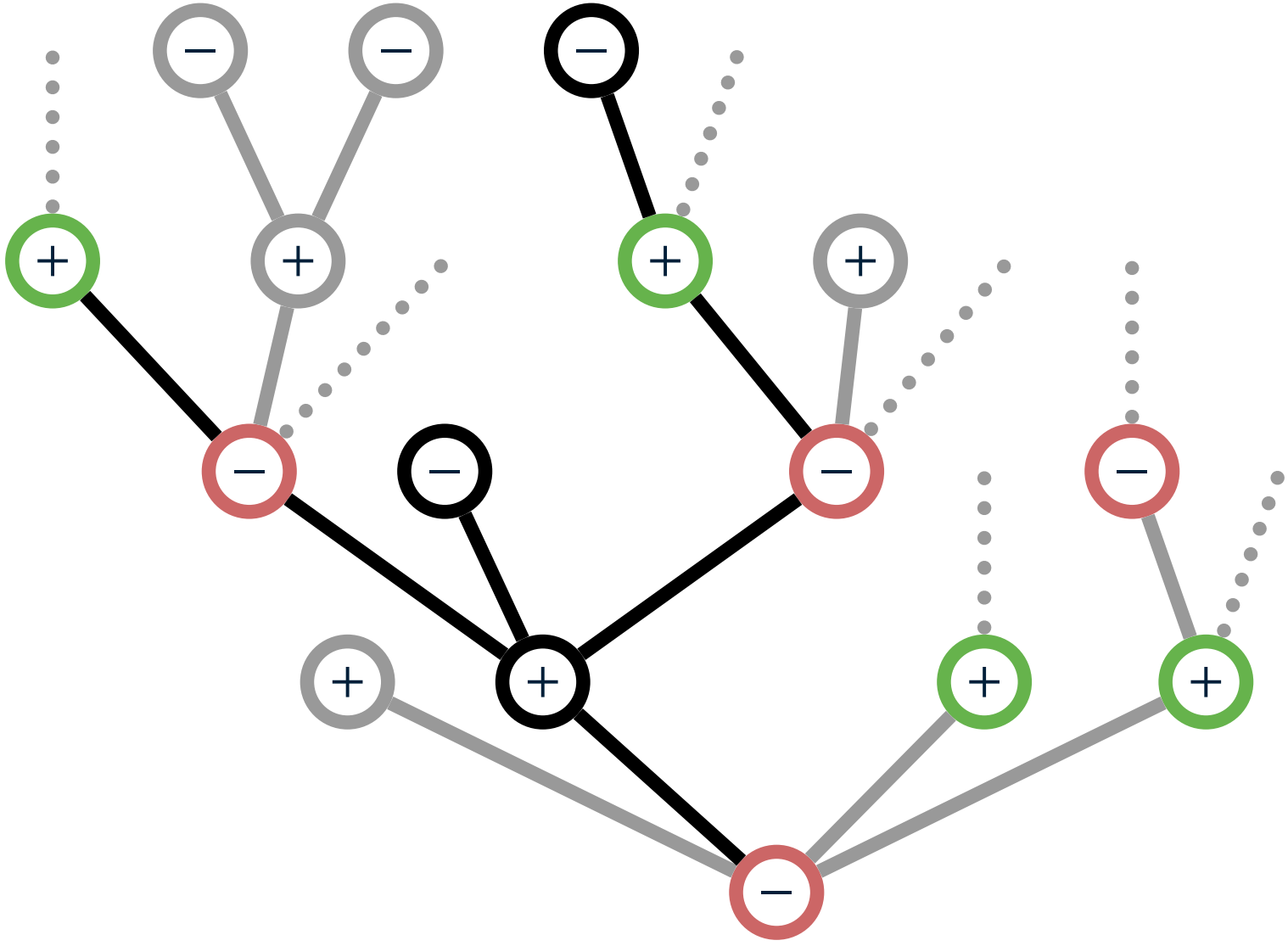
Coherence



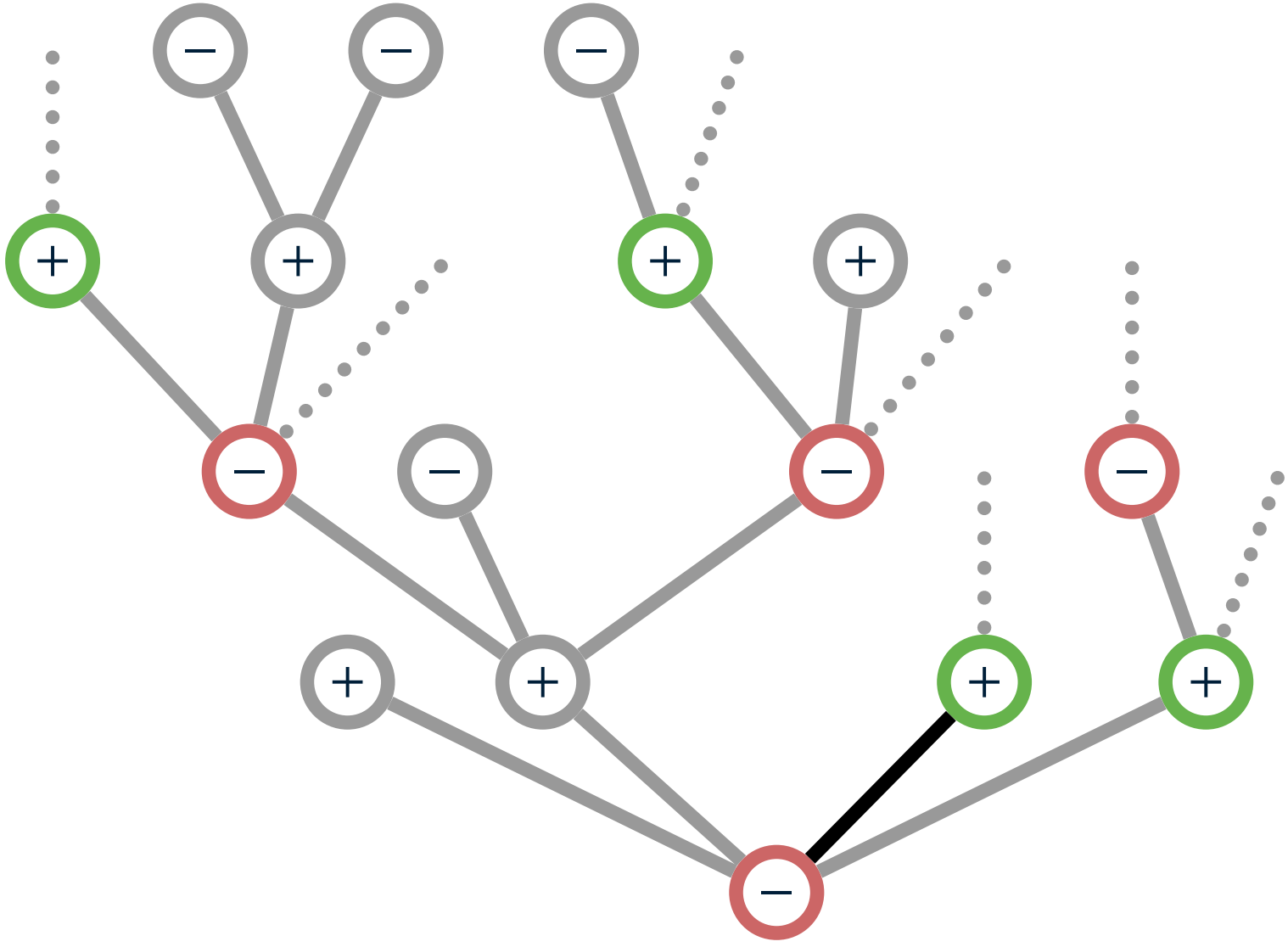
Coherence



Coherence



Coherence



What next ?

- Extension to exponentials (contraction rule)
  - additives, through $!(A \,\&\, B) \cong !A \otimes !B$, etc.
  - injectivity of statical semantics for LLP
  - simple fully complete statical semantics for LLP
- Relation with abstract Böhm trees, untyped lambda-calculus, lambda mu, etc.
- Semantics of objects calculi