MELLpol, Designs (Games) and the Relational Model

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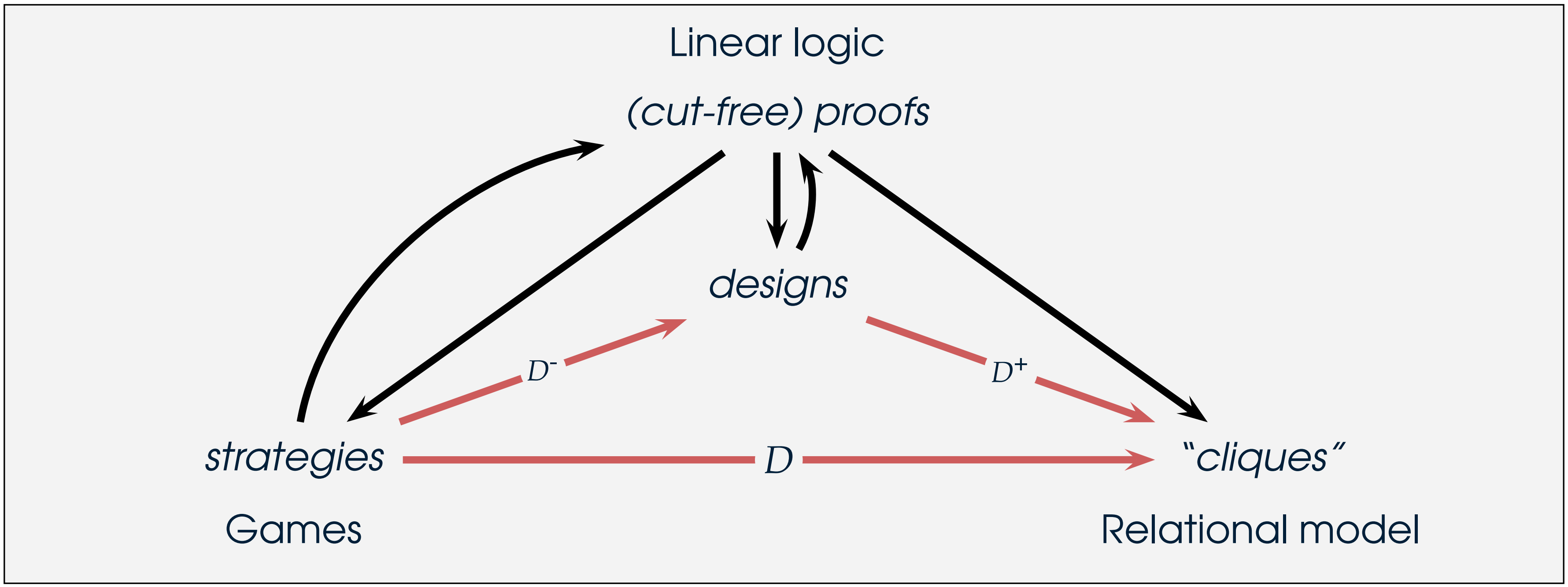
Situation



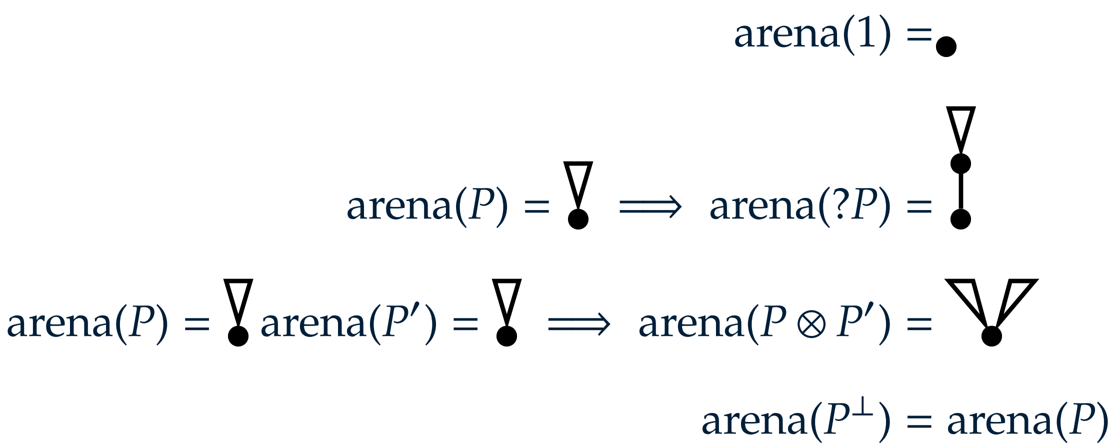
Linear logic
(cut-free) proofs
designs
D-
D+
strategies
D
"cliques"
Games
Relational model

MELLpol : formulæ

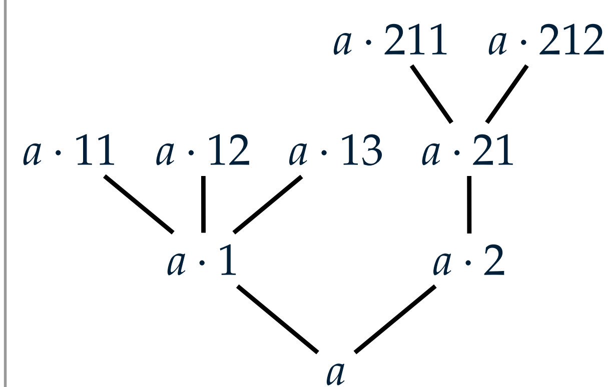
$$N := \bot \mid N \parr N \mid ?P \qquad \text{(negative formulæ)}$$

$$P := 1 \mid P \otimes P \mid !N \qquad \text{(positive formulæ)}$$

Arena of a formula :

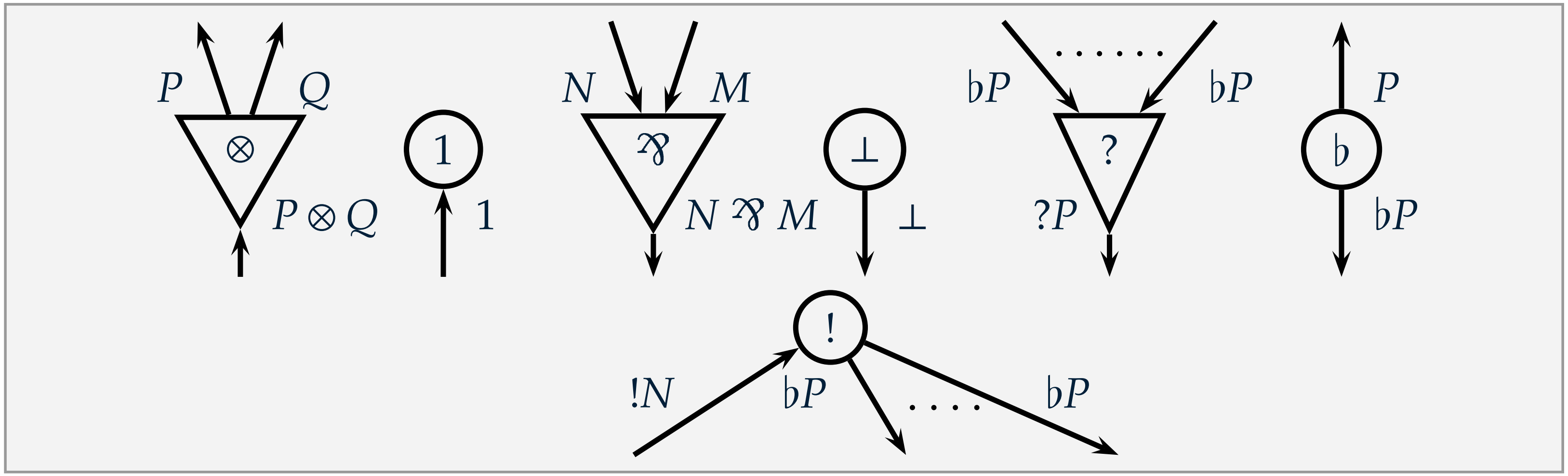


Naming :



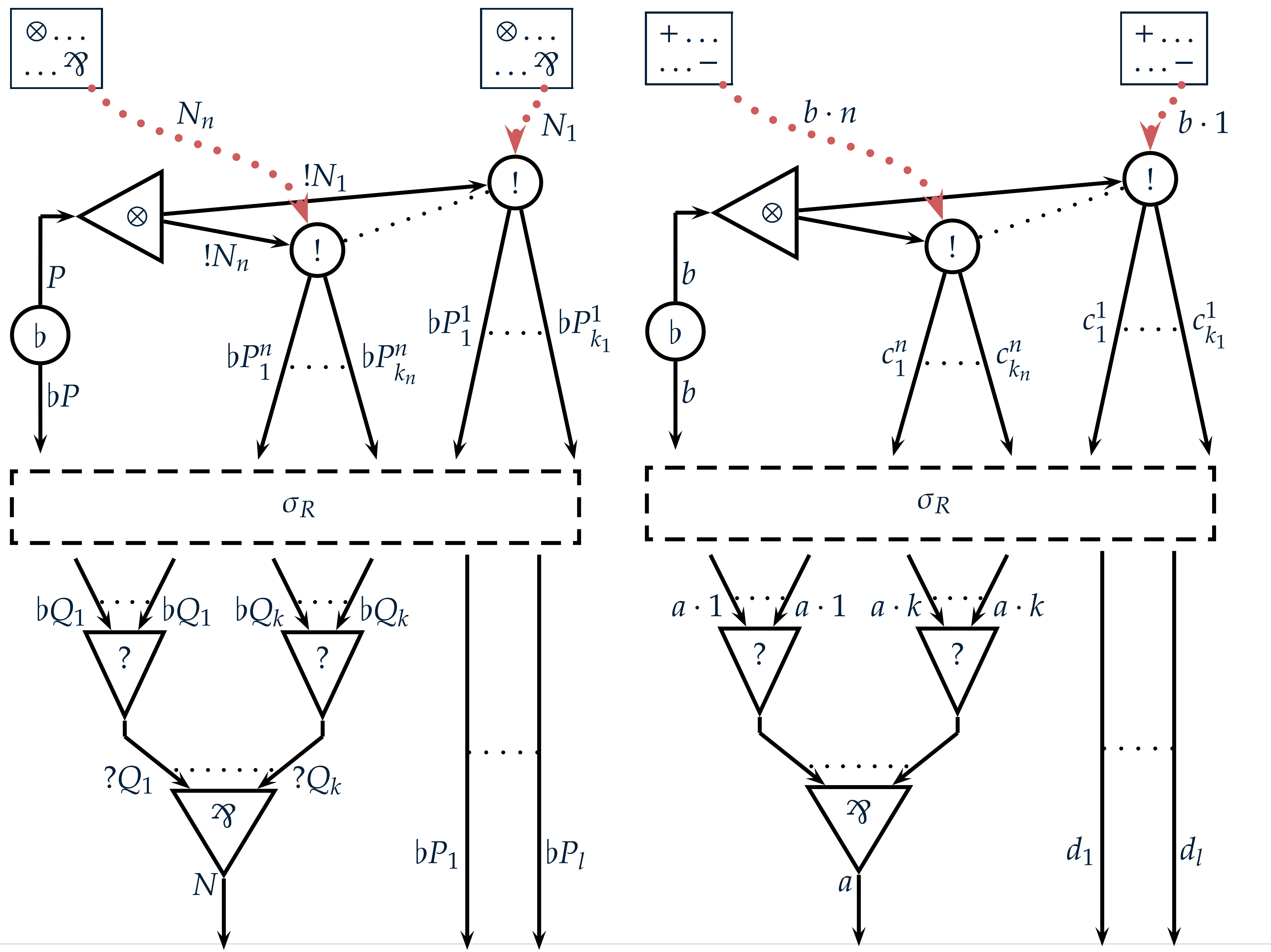
a word of integer

MELLpol : cut free proof-nets

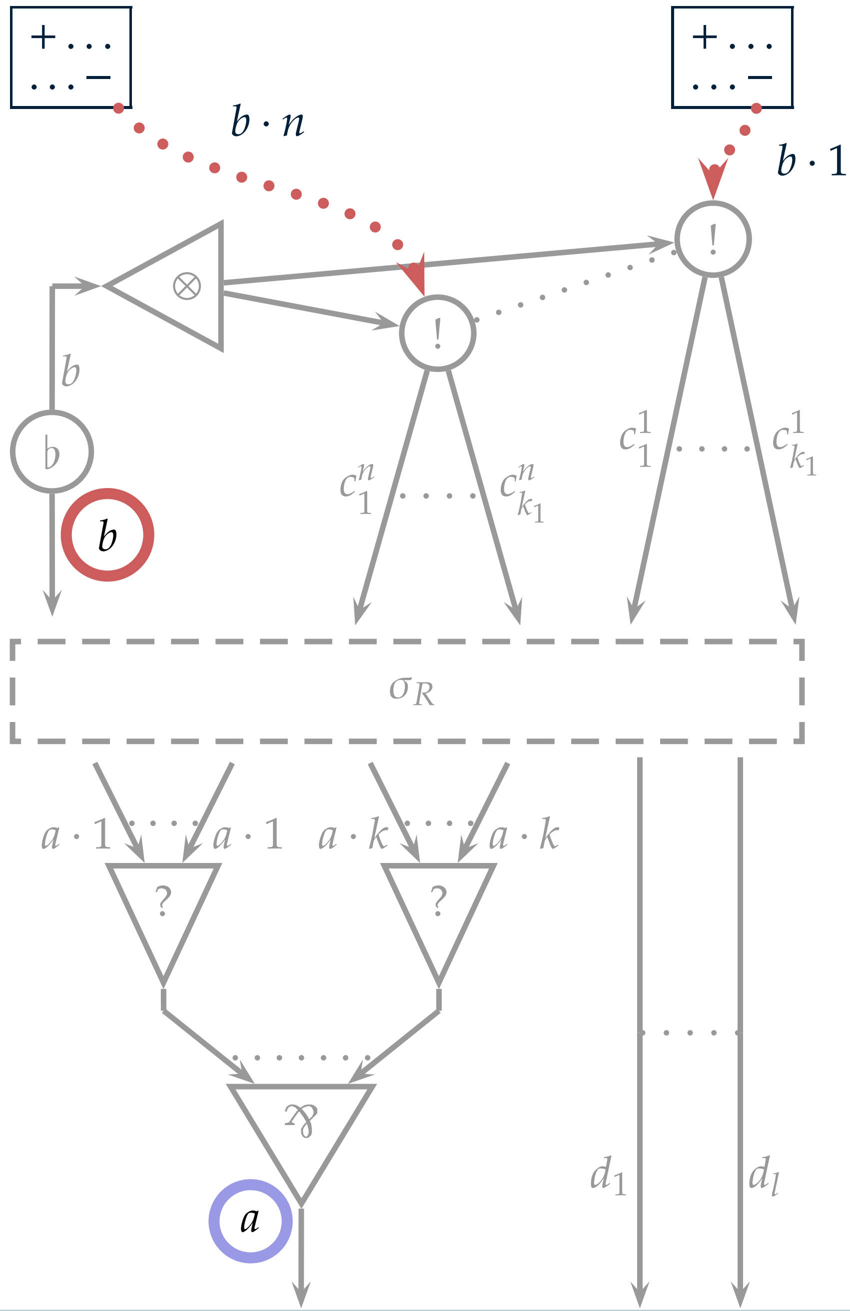


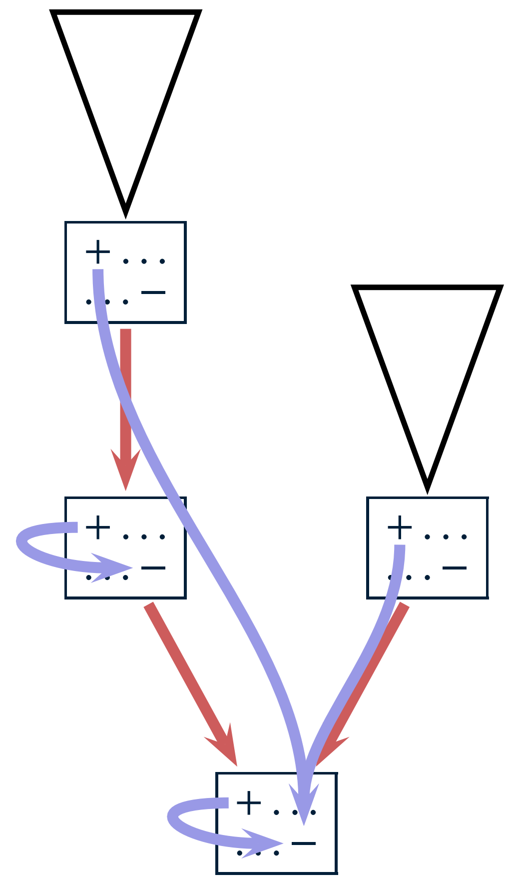
- $\flat P$ negative (not a MELLpol formula)
- Flat proof structure (input/output orientation)
- Flat proof net with negative conclusions : acyclic and exactly one $\flat$ node
- A proof net π of depth d is either :
  - a flat proof net R with no ! node ($d = 0$)
  - or a flat proof net R with, for each ! node n of conclusion $!N, \flat P_1, \ldots, \flat P_k$, a proof net π_n of conclusion $N, \flat P_1, \ldots, \flat P_k$ ($d = \max(d_n)$).

Cut free proof-nets : flat nets and naming

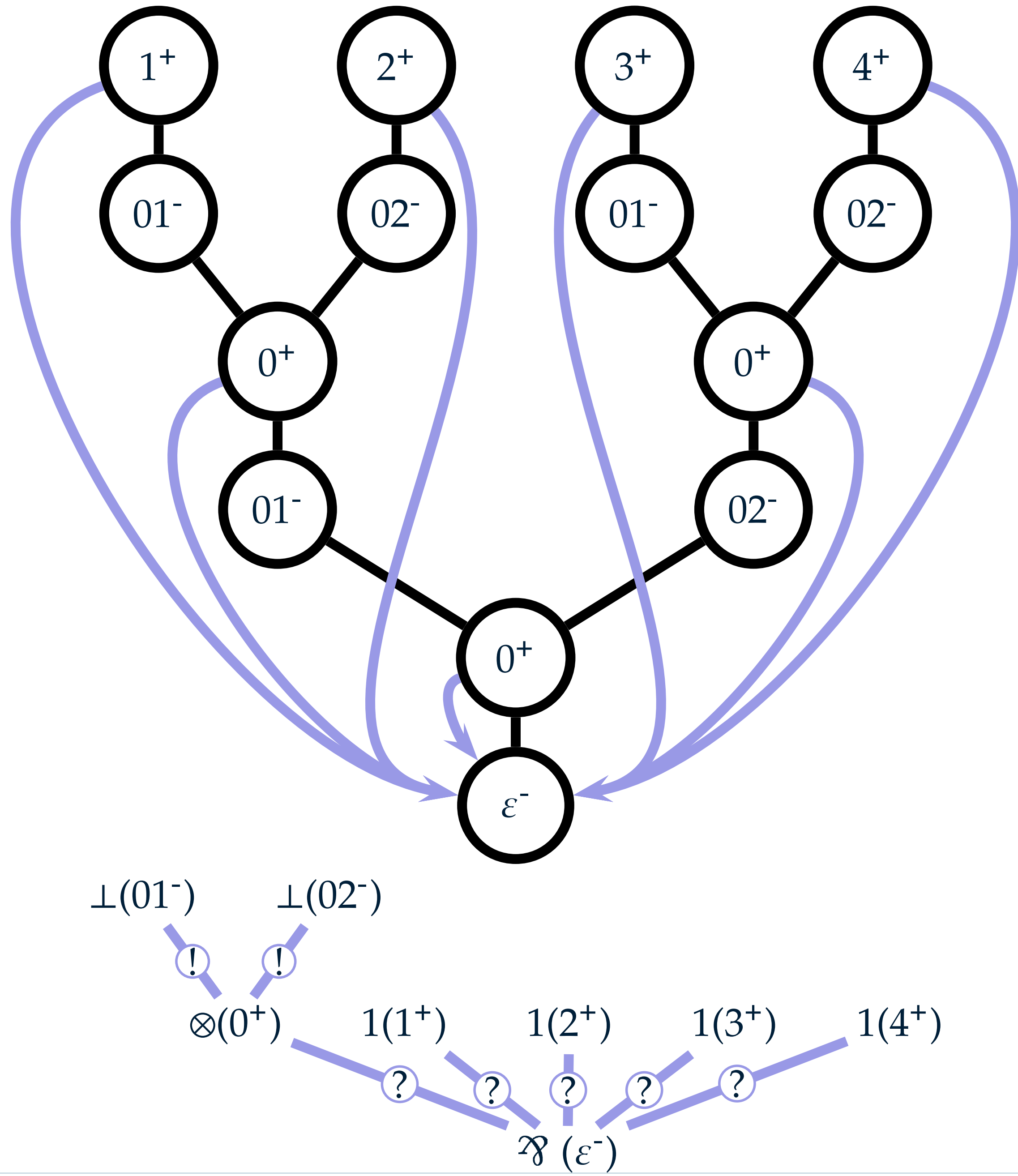


Cut free proof-nets and designs

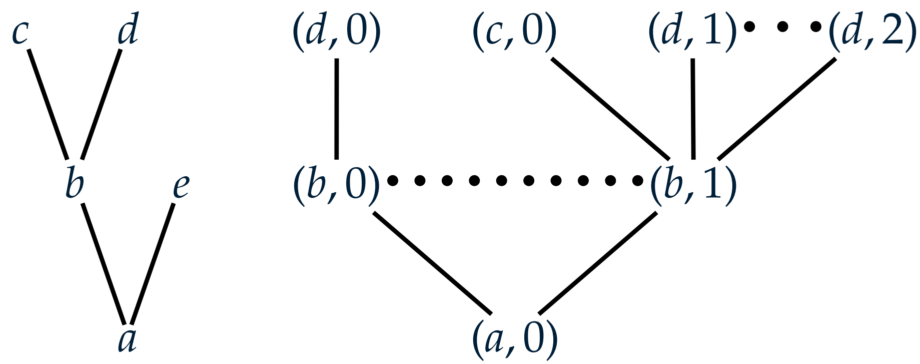




A design



Thick subtrees



- Finite trees :

$$t := (t_1, \ldots, t_n)$$

- Thick subtrees of a finite tree $t = (t_1, \ldots, t_n)$:

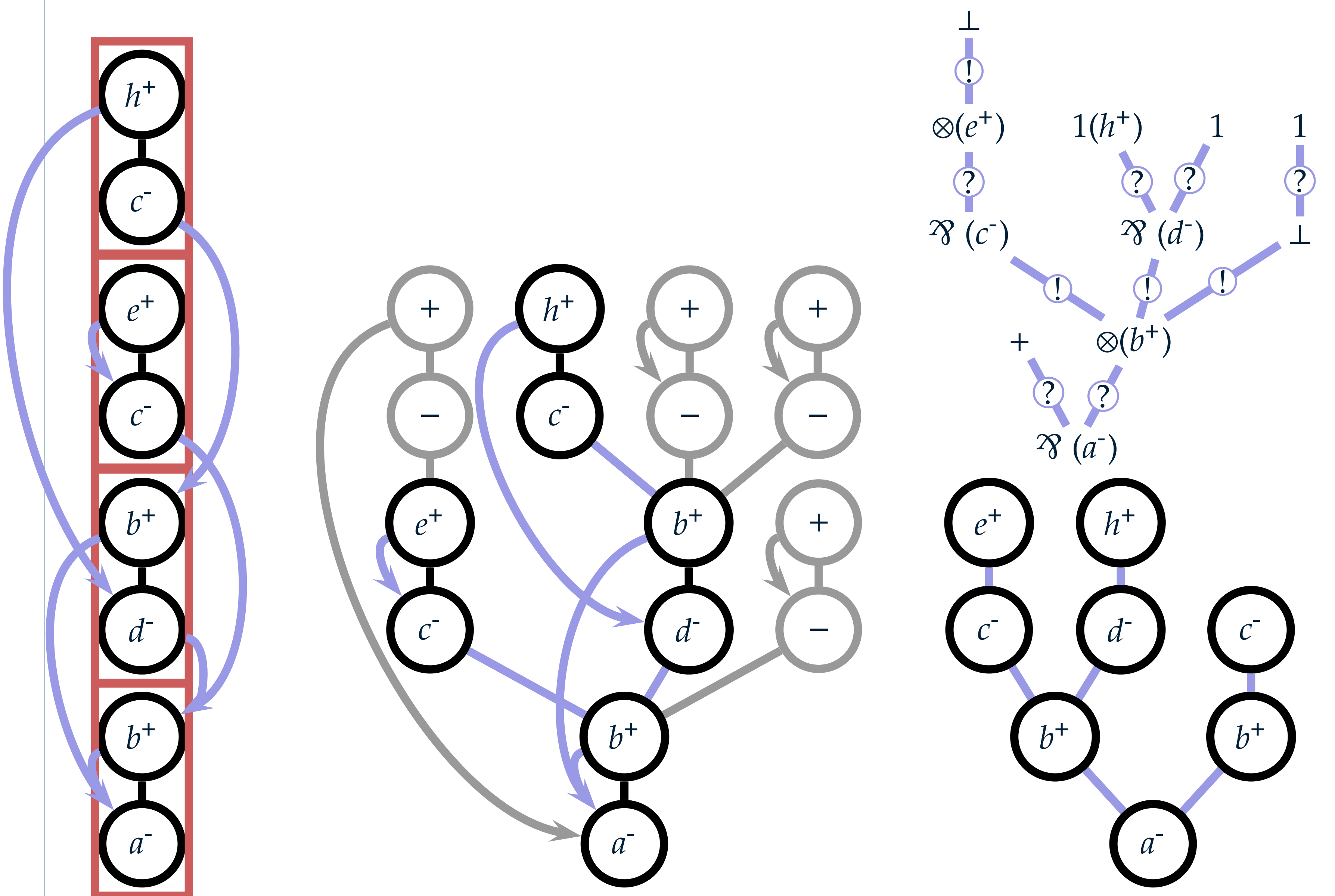
$$\tau := (\mu_1, \ldots, \mu_n) \quad \mu_i \text{ finite multiset of thick subtrees of } t_i$$

- there is no empty thick subtree
- First application :

$$[A]_{\mathsf{rel}} = |A| = \mathrm{TST}(\mathrm{arena}(A))$$

$$|!N_1 \otimes !N_n| = \mathcal{M}_{\mathsf{fin}}(|N_1|) \times \ldots \times \mathcal{M}_{\mathsf{fin}}(|N_n|)$$

Play (cut) Design (proof) Point (Formula)

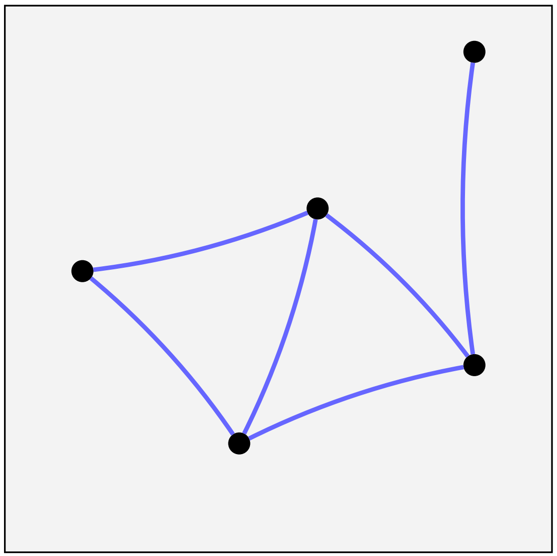


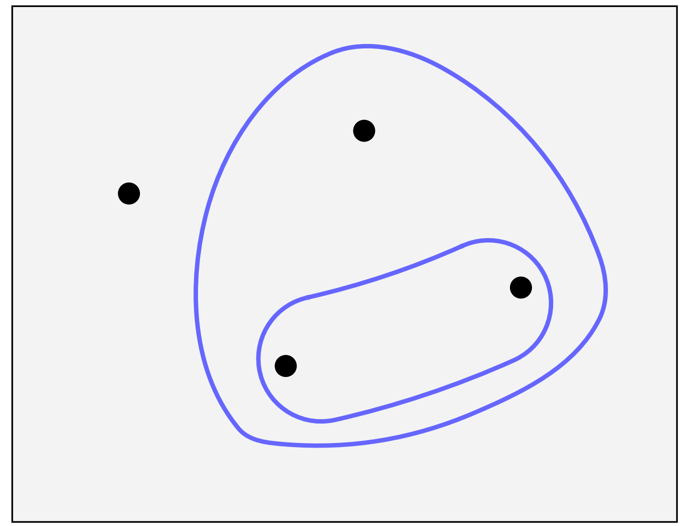
Strategy ϕ

$D^-(\phi) = \mathrm{TST}(\mathcal{D}_\pi)$

$[\pi]_{\mathsf{Rel}} = D^+(\mathrm{TST}(\mathcal{D}_\pi))$

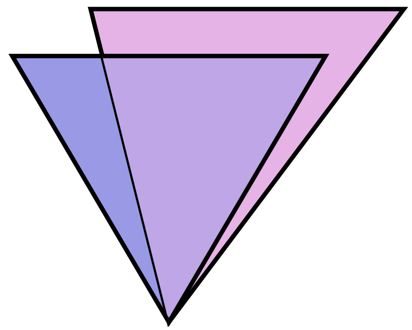
Coherence and hypercoherence spaces





Coherence in MALpol

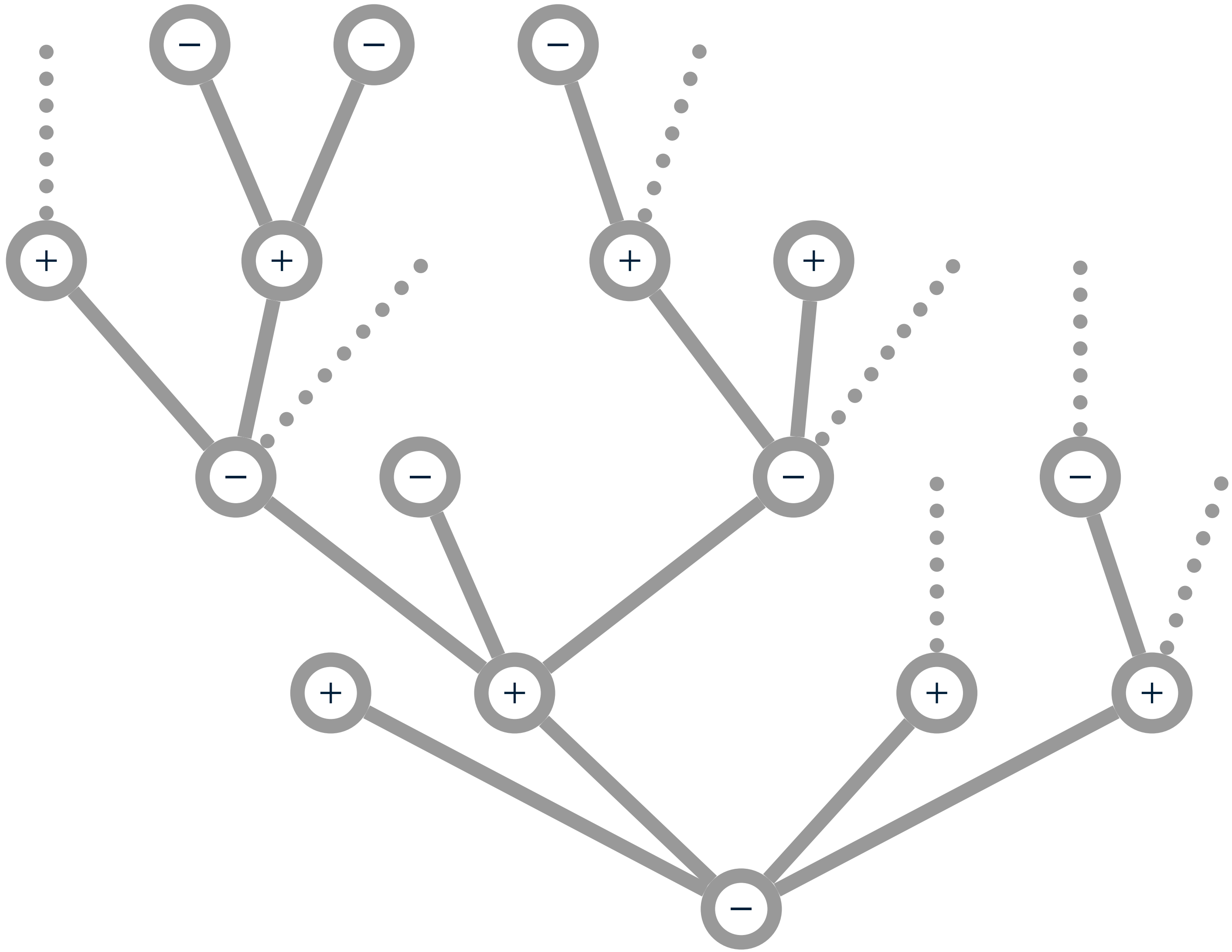
- Coherence of $p_0, \ldots, p_n$ depends only on $\cap p_i$ (intersection) and $\cup p_i$ (surperposition) and is equivalent to the coherence of $\cap p_i$ and $\cup p_i$.



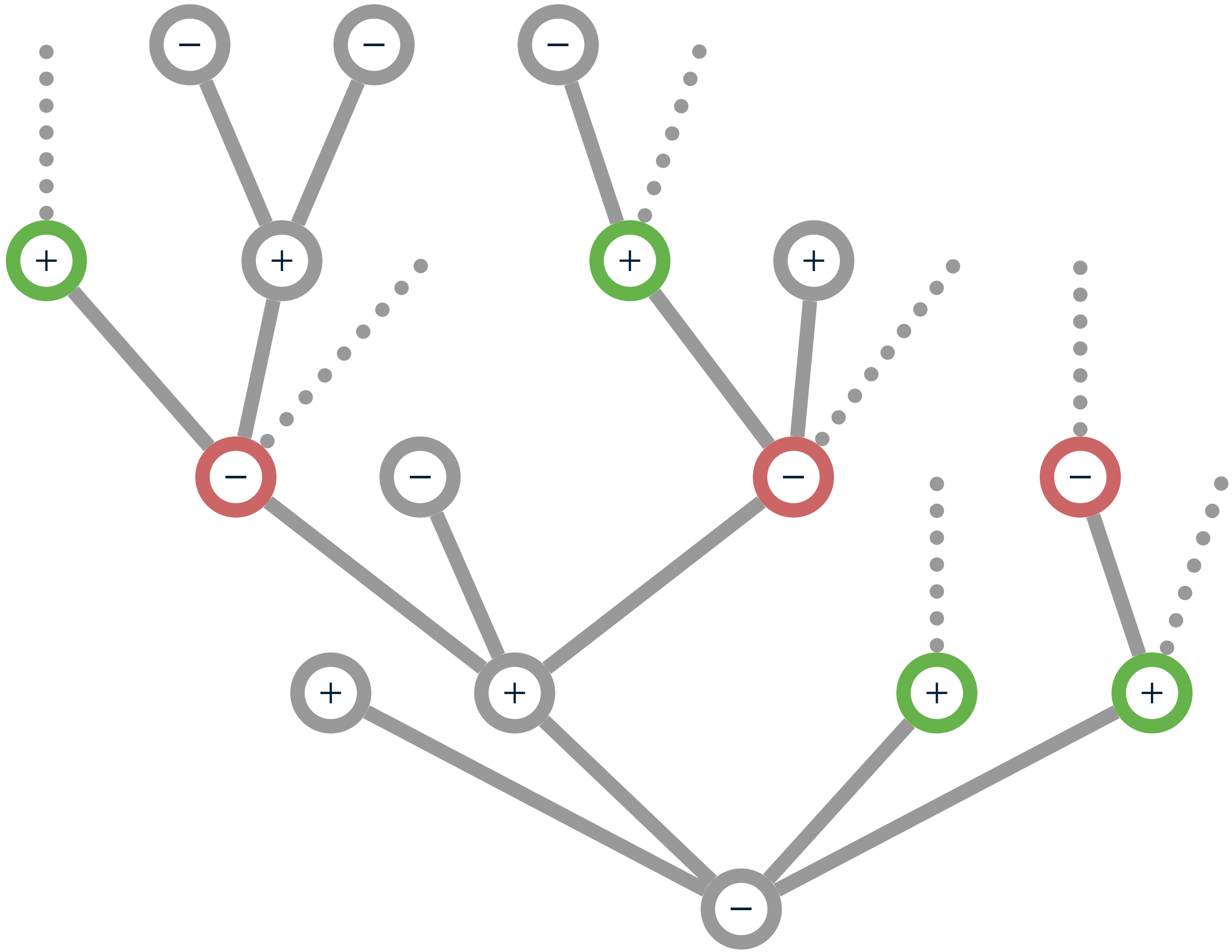
- Interpretations of proofs are closed by intersection and superposition of trees/points.

(. . .)

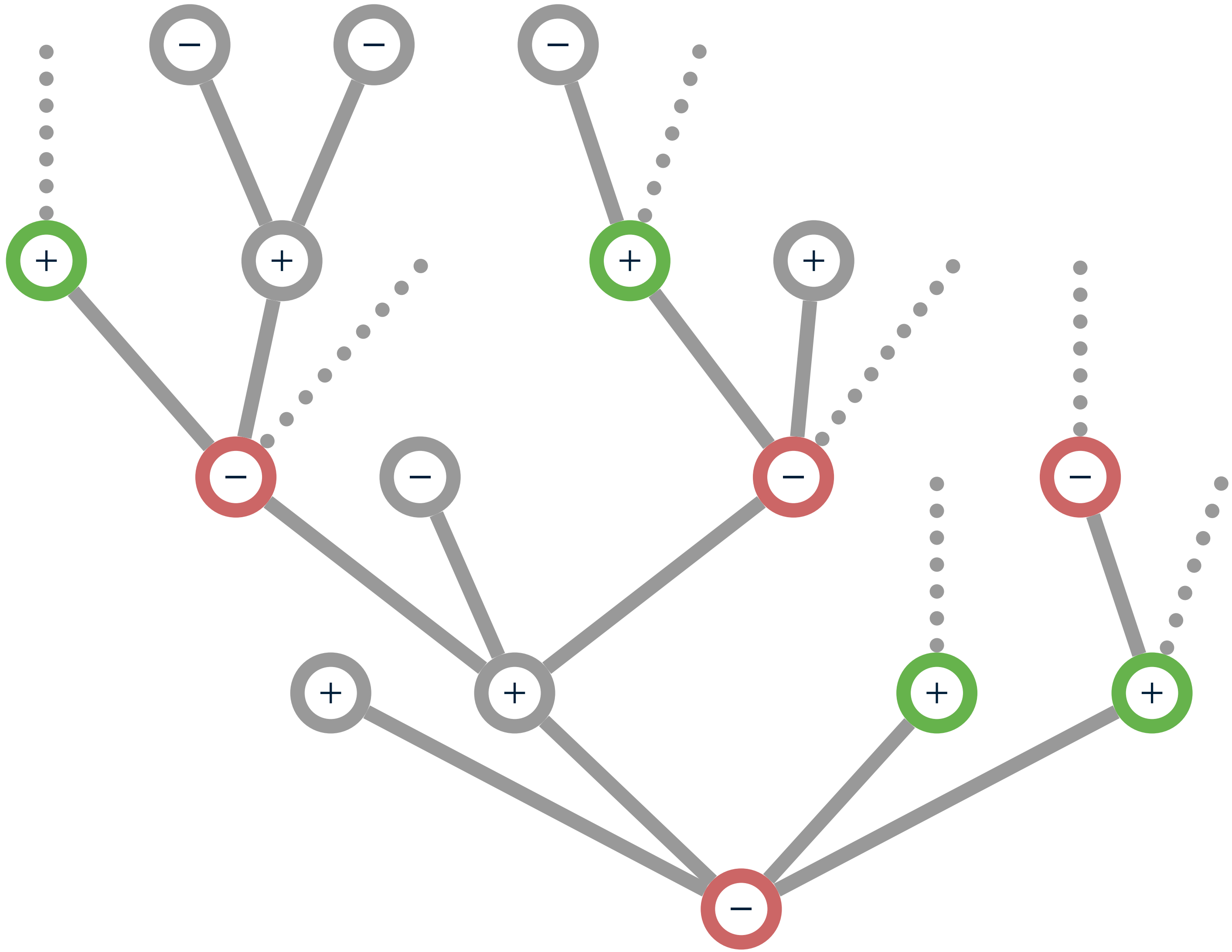
Coherence in MALpol



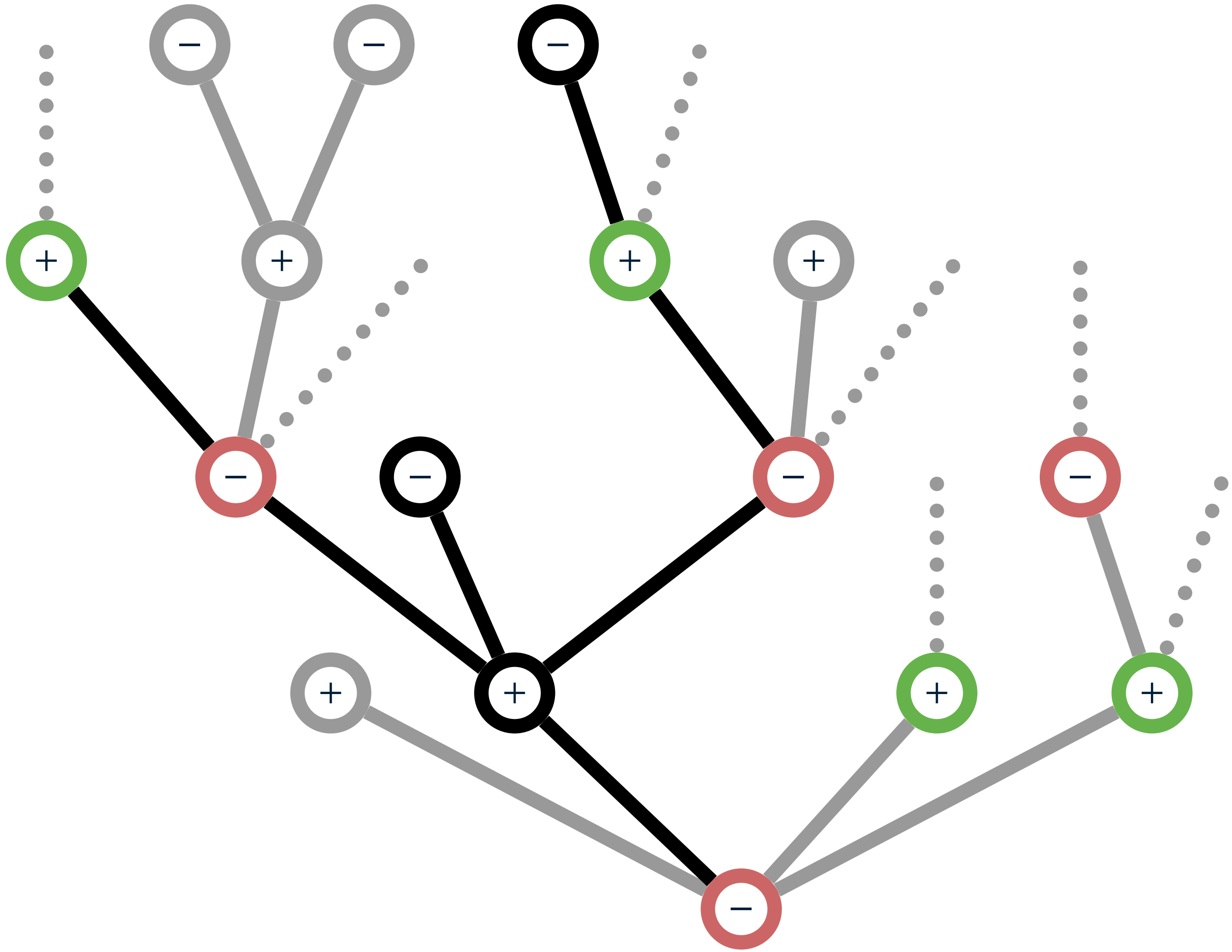
Coherence in MALpol



Coherence in MALpol



Coherence in MALpol



Coherence in MALpol