Non-uniform Hypercoherences

Pierre Boudes

Institut de Mathématiques de Luminy

What's this talk about?

Denotational semantics (of λ-calculus, PCF, LL)

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- complexity issue (ELL, LLL), polarities, ...

Uniformity at a first sight

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$$\texttt{P} = \lambda b.\ \ \texttt{if}\ b\ \ \texttt{then}\ \{\ \texttt{if}\ b\ \texttt{then}\ t_1\ \texttt{else}\ t_2\ \}$$
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Coherence spaces and **hypercoherences** semantics (set-based) are *uniform*

$$P^{\bullet} = \{(\{\texttt{t}\}, t_1^{\bullet}), (\{\texttt{f}\}, t_4^{\bullet})\}$$

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Relational semantics is *non-uniform*

$$P^{\bullet} = \{([\texttt{t},\texttt{t}], t_1^{\bullet}), ([\texttt{t},\texttt{f}], t_2^{\bullet}), ([\texttt{t},\texttt{f}], t_3^{\bullet}), ([\texttt{f},\texttt{f}], t_4^{\bullet})\}$$

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Multiset-based coherence spaces and hypercoherences semantics are also *uniform*

$$P^\bullet = \{([\texttt{t}, \texttt{t}], t_1^\bullet), ([\texttt{f}, \texttt{f}], t_4^\bullet)\}$$

Denotational semantics

Static semantics

Dynamic semantics

Denotational semantics

Static semantics: Agents are sets of **points**

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uniformity

Denotational semantics

Static semantics

Dynamic semantics

Denotational semantics

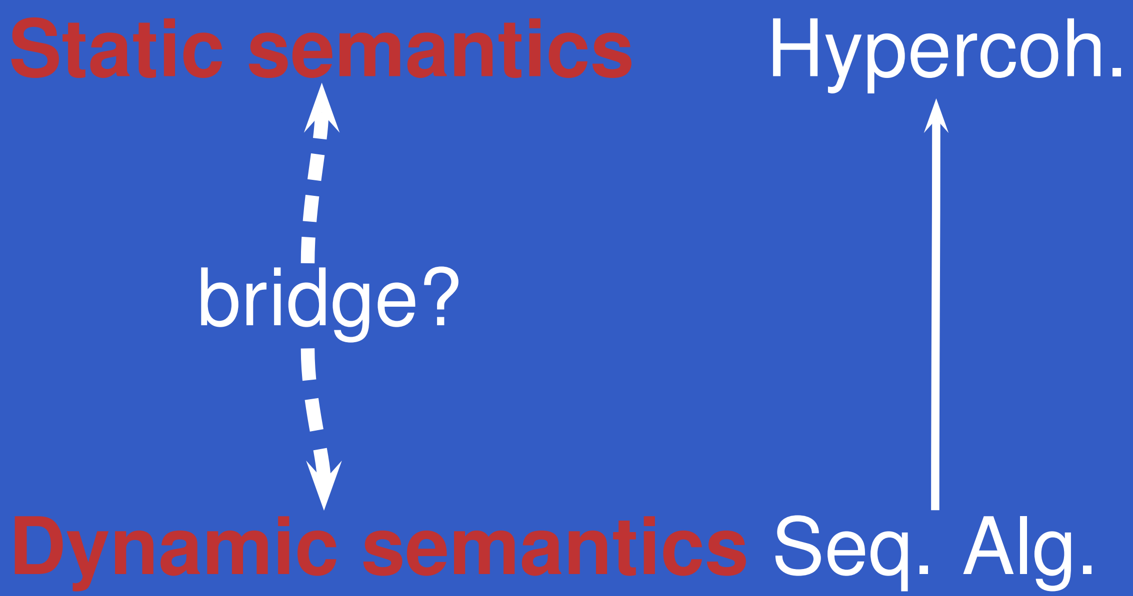
Static semantics Hypercoh.

bridge?

Ext. Coll. EHRHARD [Ehr96]

Dynamic semantics Seq. Alg.

Denotational semantics

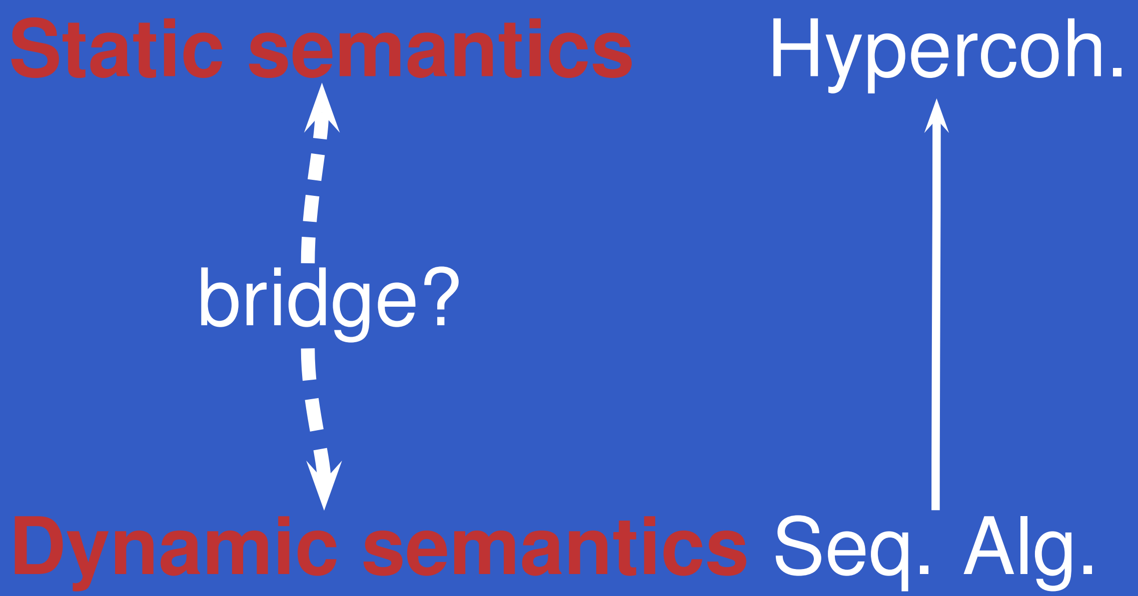


PER $\sim$. Equality on basis types and :

$$f \sim_{A \to B} g \text{ iff } x \sim_A y \implies f(x) \sim_B f(y) \quad (\forall x, y)$$

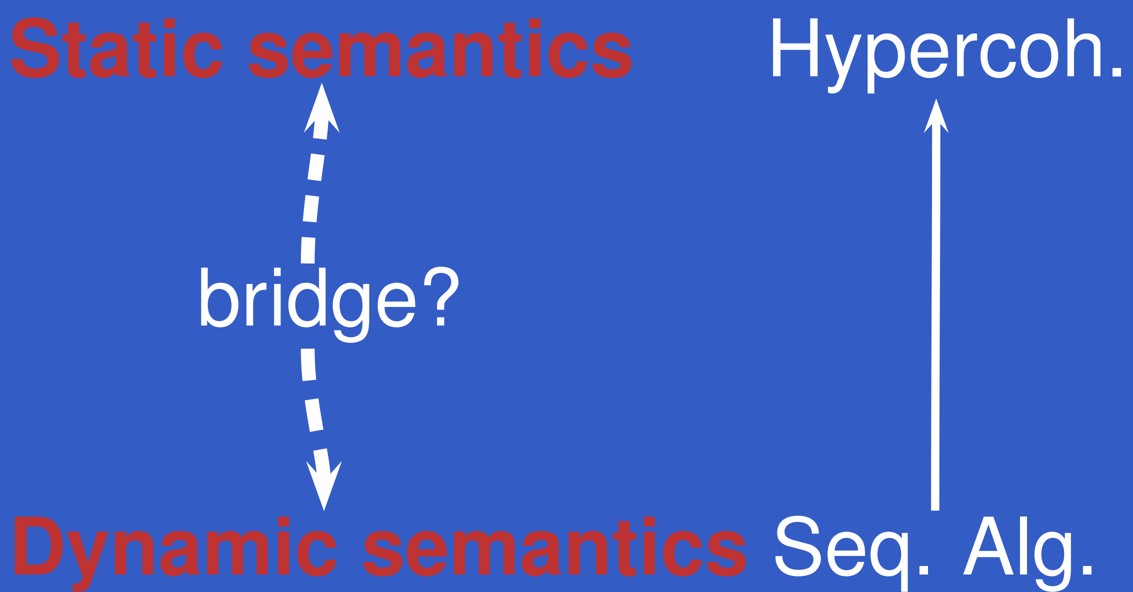
The extensional collapse of a model M is $M/\sim$.

Denotational semantics



- one-way bridge

Denotational semantics



- one-way bridge
- abstract result

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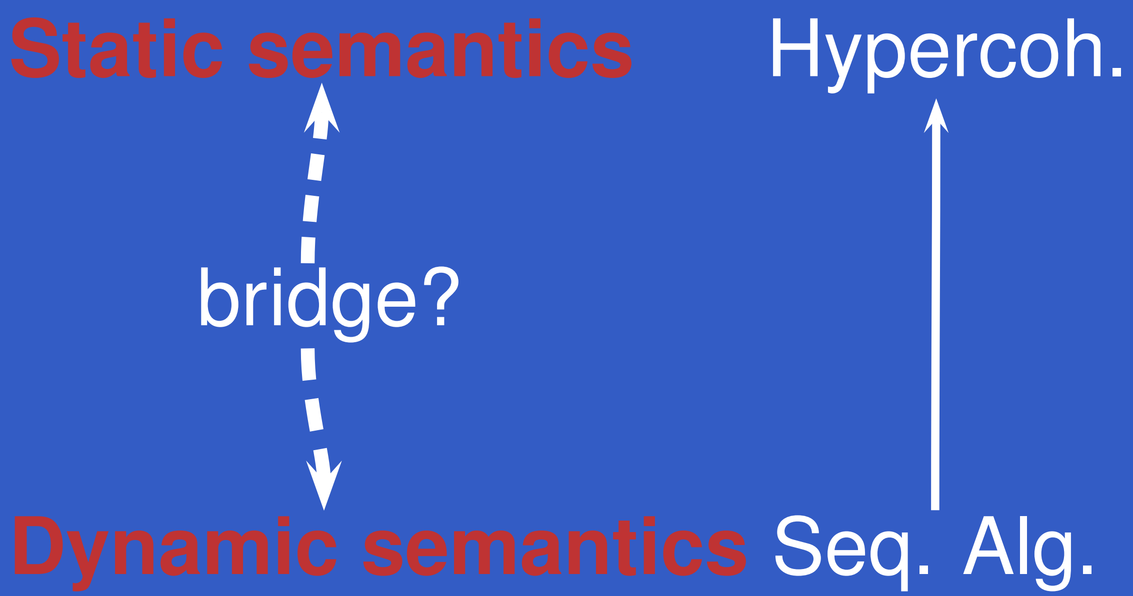
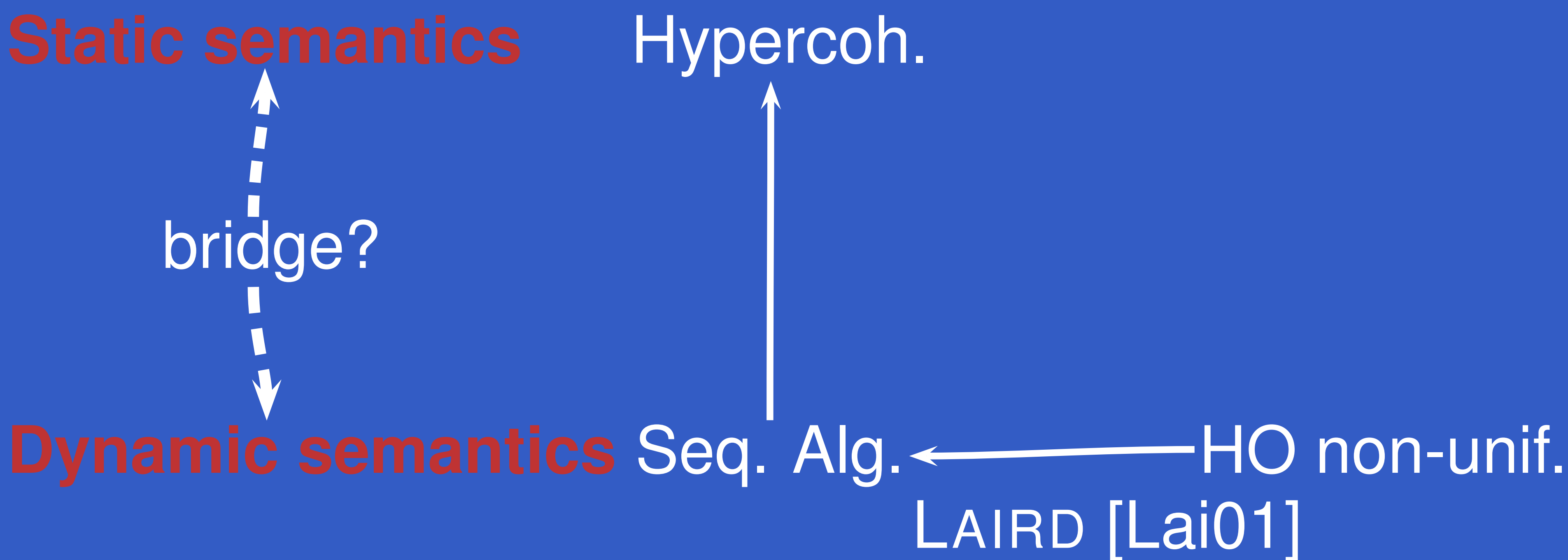


- one-way bridge
- abstract result
- Only simple types. Not the full power of linear logic.

Denotational semantics



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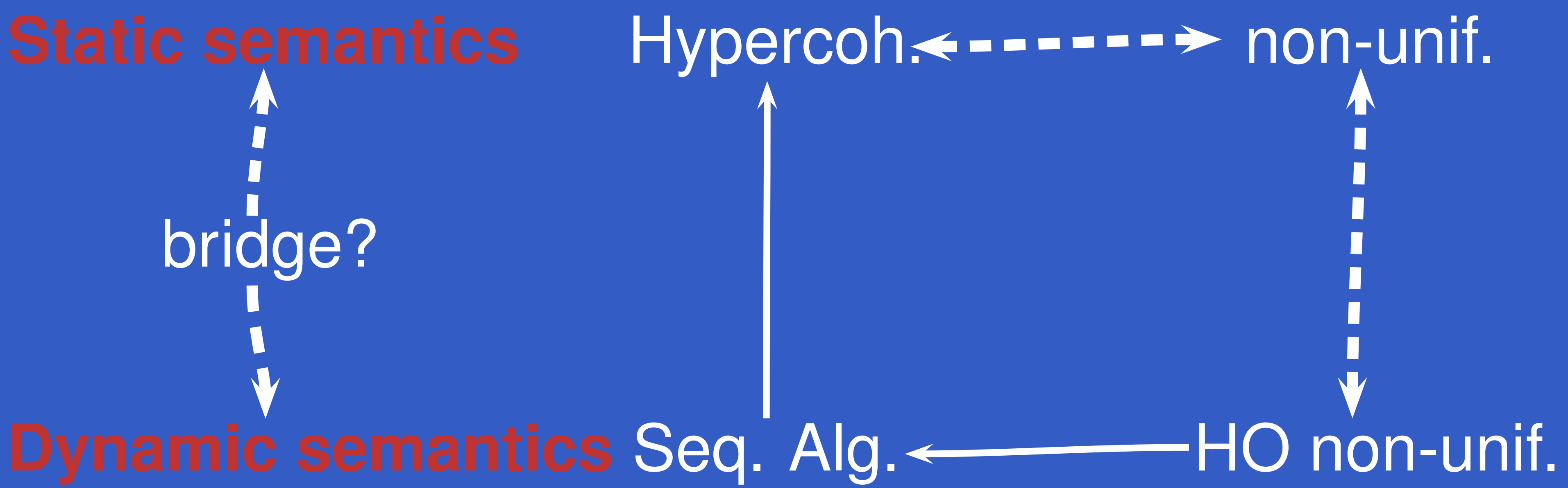

Shifting to non-uniformity might improve the bridge

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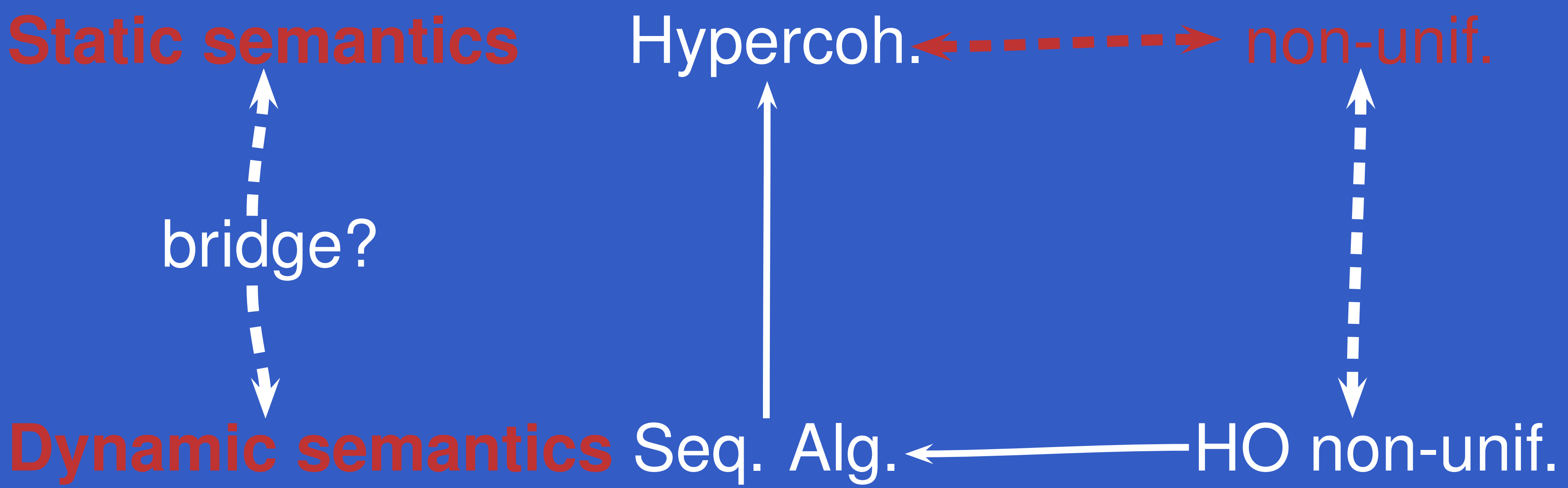

Shifting to non-uniformity might improve the bridge

Uniform spaces

Uniform spaces

- Coherence space $X = (|X|, \coh_X)$: graph

$$\{[a,a] \mid a \in |X|\} \subseteq \coh_X \subseteq \mathcal{M}_{\{2\}}(|X|)$$

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Powers

A **Power** P is a functor from the category of sets and inclusions to itself.

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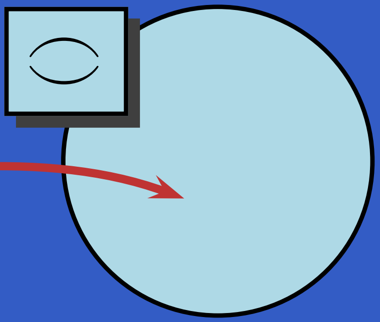

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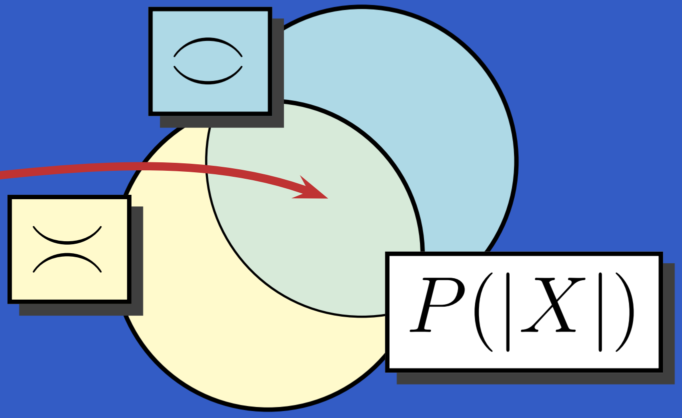

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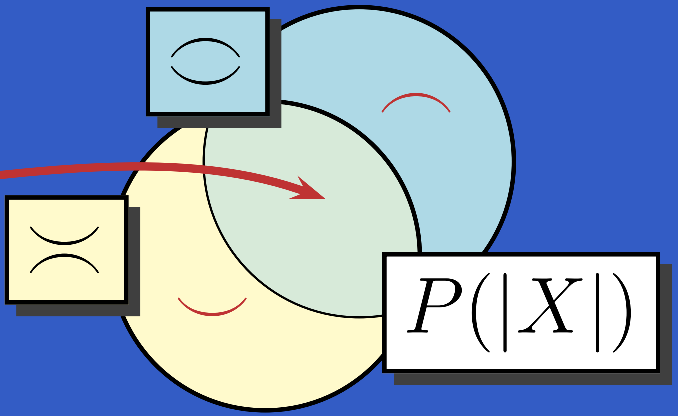

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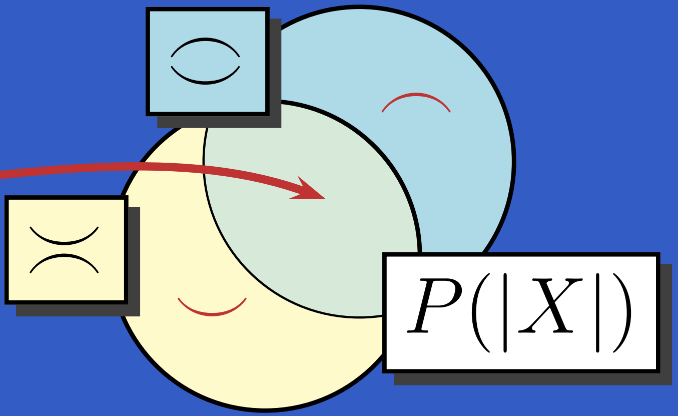

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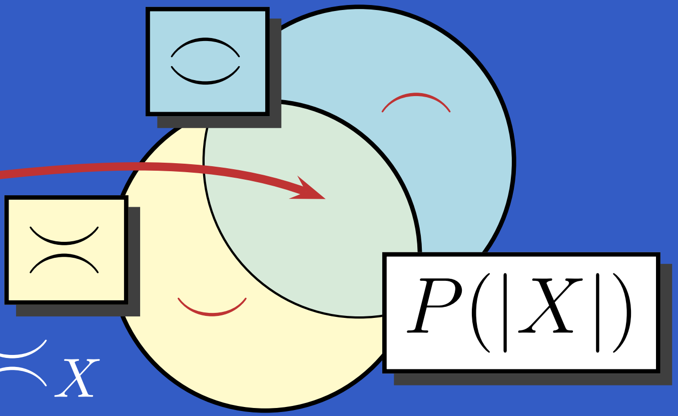

- **Agent** $x \subseteq |X|, P(x) \subseteq \mathrel{\overset{\frown}{\smile}}_X$
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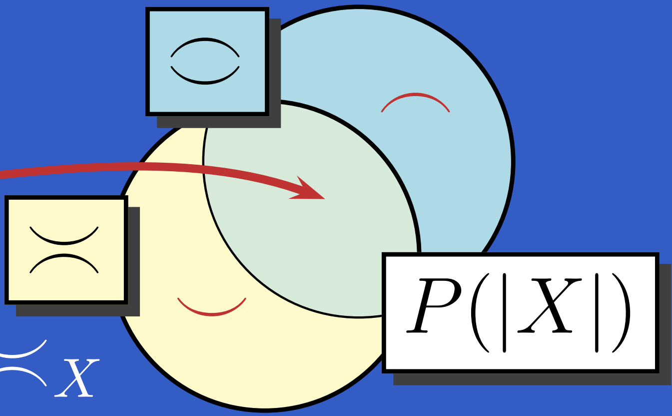

- **Agent** $x \subseteq |X|, P(x) \subseteq \coh_X$
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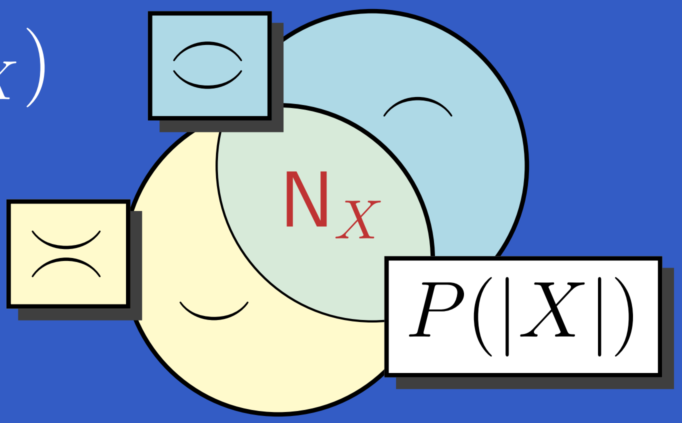
- **Determinism** $\sharp(x \cap y) \leq 1$

Non-uniform spaces

P-coherence space $(|X|, \overset{\frown}{\smile}_X, \asymp_X)$



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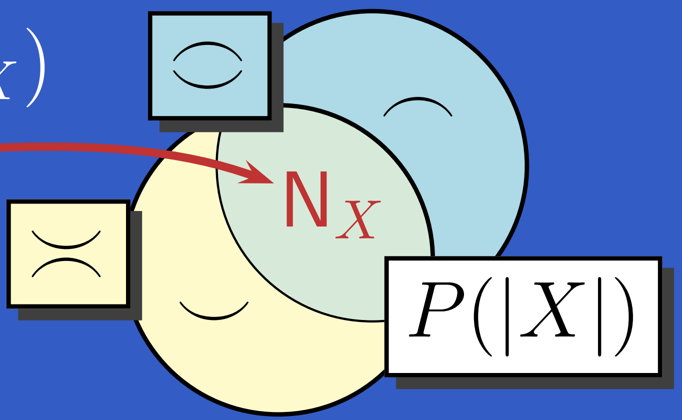
No longer reflexive. Neutrality.



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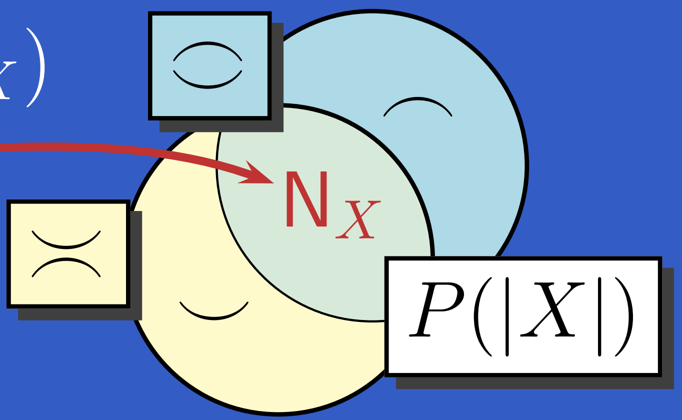

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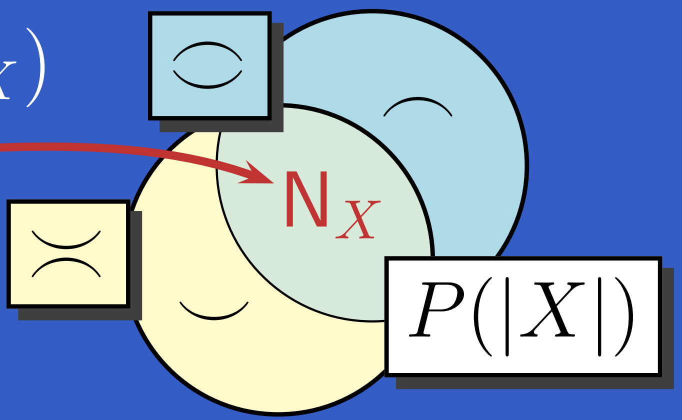

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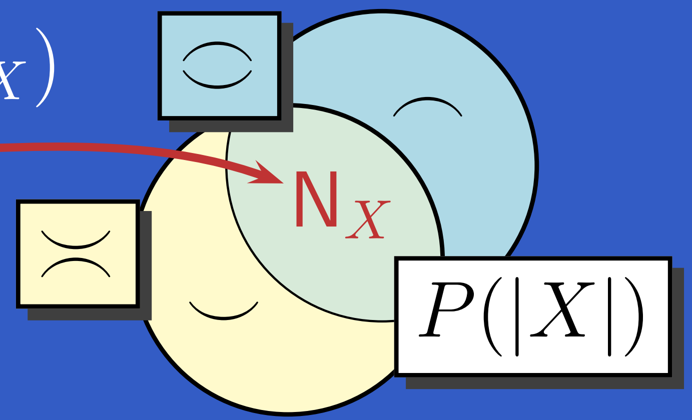

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$\rightarrow$ a class of non-uniform coherence semantics which we describe now using the Power $\mathcal{M}_K$ (multisets whose cardinalities are in K) where $K \subseteq \mathbb{N} \backslash \{0, 1\}$.

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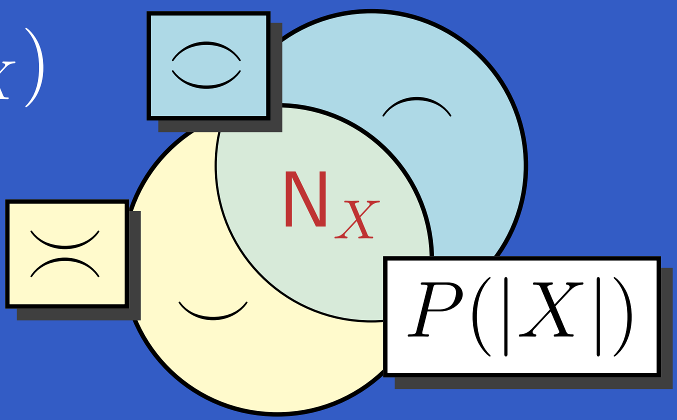
$\mathcal{M}_K$-coherence space $(|X|, \frown_X, \asymp_X)$



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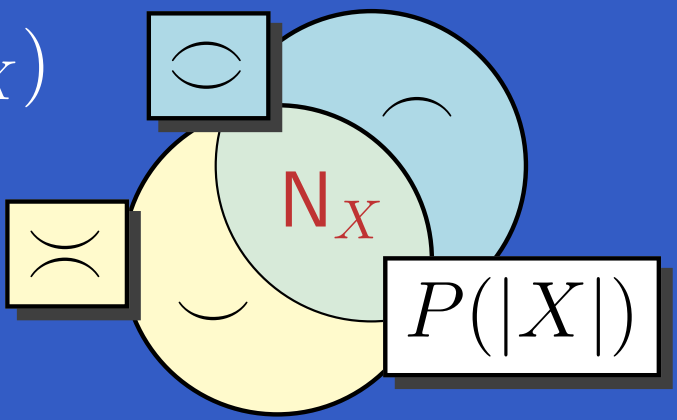

- MALL model: standard pattern

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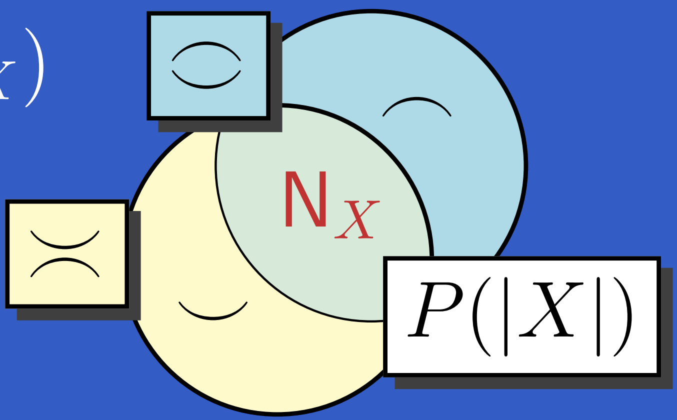

- MALL model: standard pattern

eg, $|X \multimap Y| = |X| \times |Y|$ and, for $s \in \mathcal{M}_K(|X \multimap Y|)$:

$$s \in \overset{\frown}{\smile}_{X \multimap Y} \text{ iff } \begin{cases} \pi_1(s) \in \overset{\frown}{\smile}_X \implies \pi_2(s) \in \overset{\frown}{\smile}_Y \\ \pi_1(s) \in \asymp_X \Longleftarrow \pi_2(s) \in \asymp_Y \end{cases}$$

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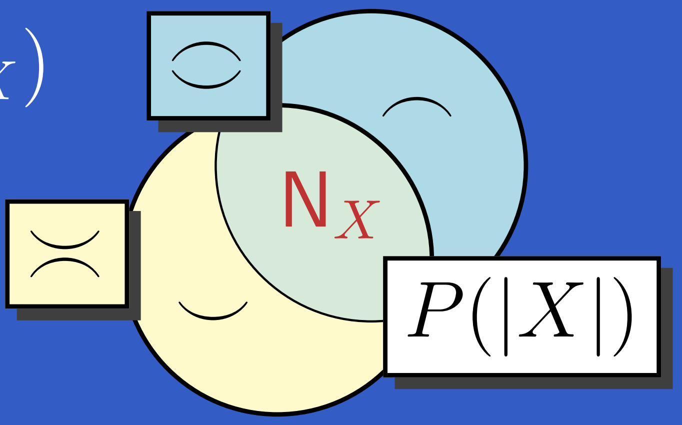

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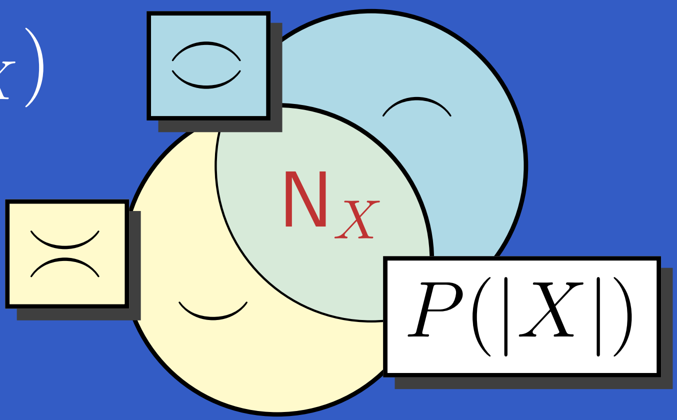

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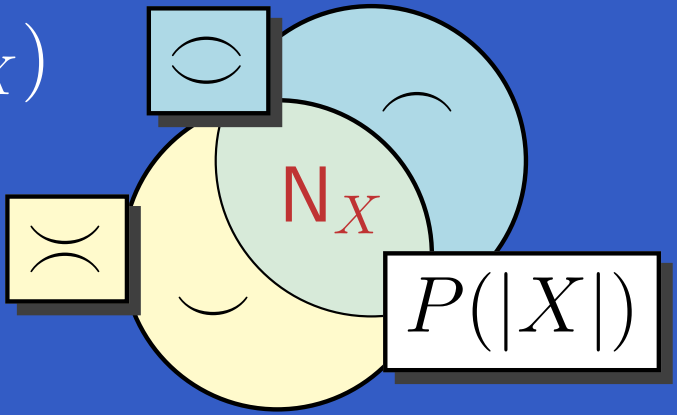

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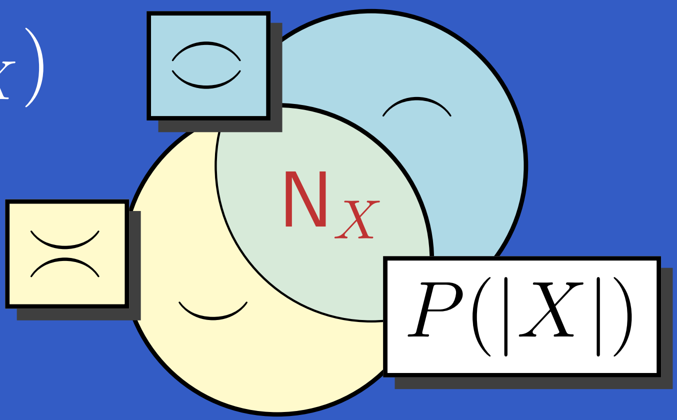

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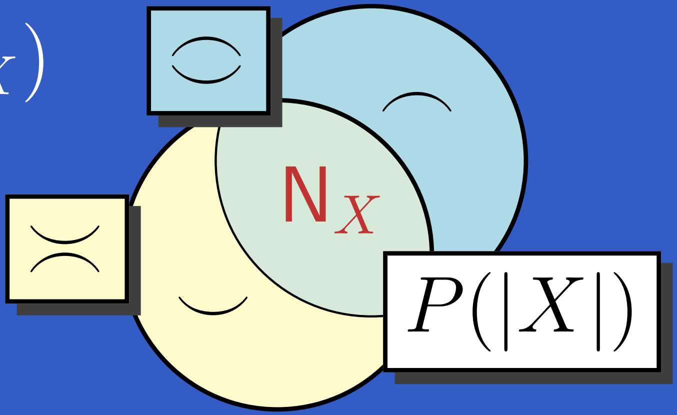

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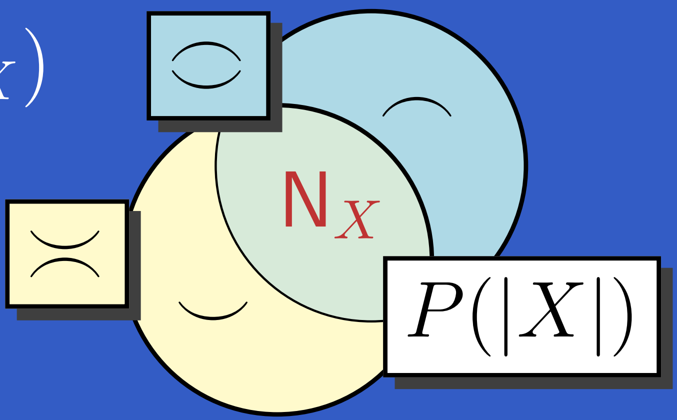

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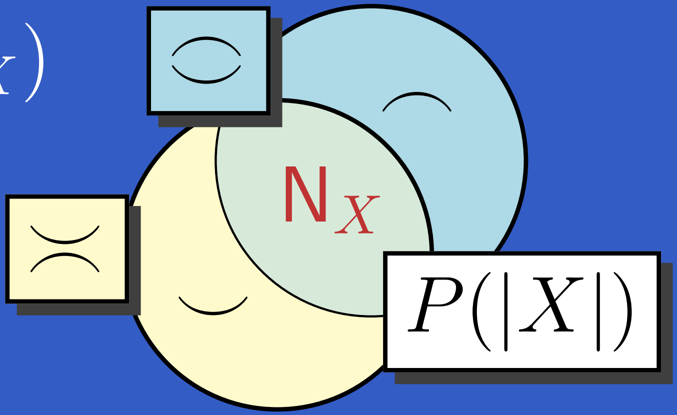

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- Hopefully, we have a better solution!

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This web is the same as for relational model, so the semantics is non-uniform. A uniform web would have been $\{x \in \mathcal{M}_{\text{fin}}(|X|) \mid \mathrm{supp}(x) \in \text{agent}(X)\}$.

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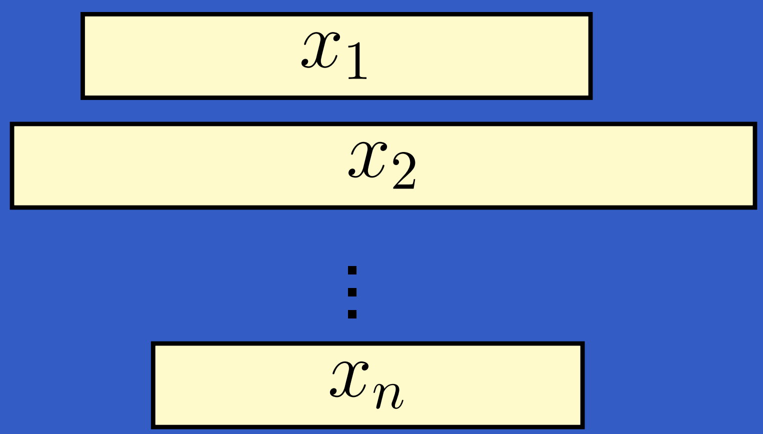

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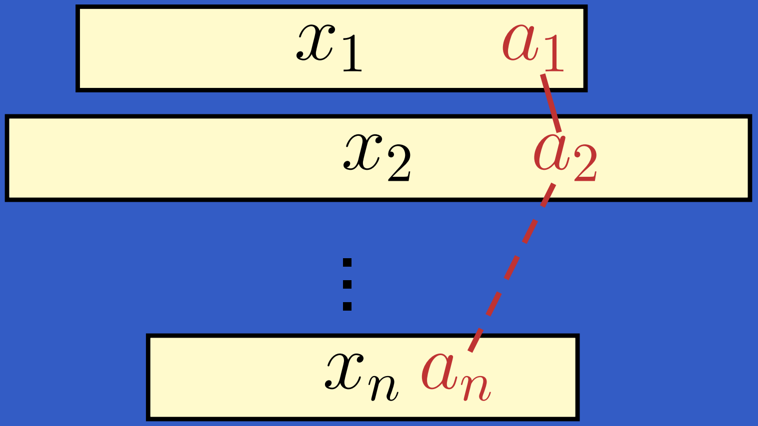

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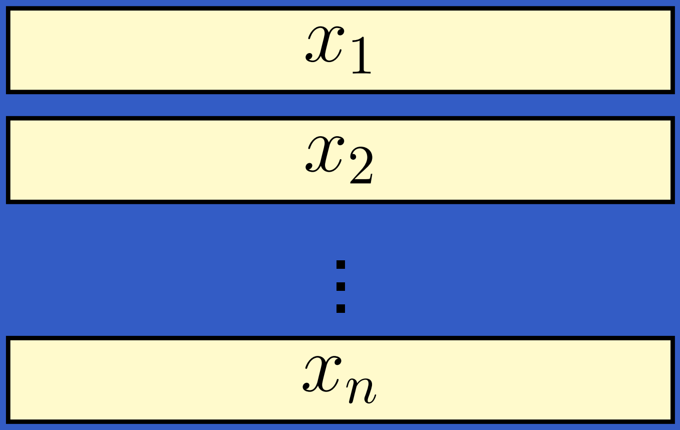

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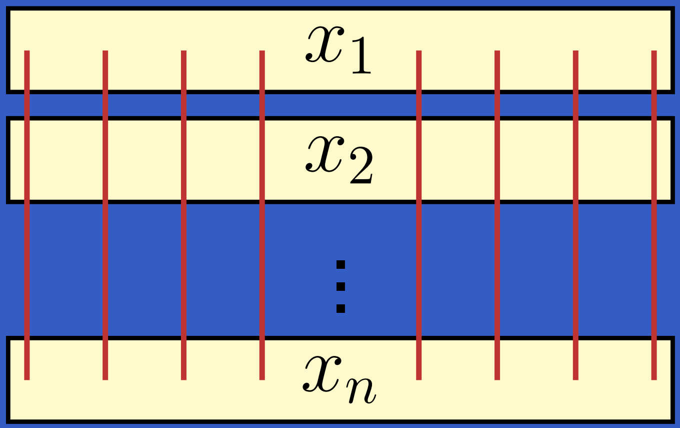

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- $\underset{u}{!} = N_K!$. **A new class of uniform models** related to the non-uniform ones in a very comfortable way.

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- Multicoherences $\neq$ hypercoherences $\longrightarrow$ two (extensionally) different notions of higher order sequentiality. (Contrarily to what was expected, *eg* in [Lon02]).

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