Extensional filters reveal algorithms

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Introduction

Motivations

- Are "complicated" programming constructions really useful? We're still Turing-complete without them. High-order? Non-determinism? Co-arity? Multiple tapes? Large alphabets? ...
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Expressivity and programming languages

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High order is "more expressive" than first order, but Turing-completeness hide this.

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- Programming language: $\mathtt{Pgms}$ $\supset \mathtt{L}, \mathtt{M}$ (syntactic)

$\mathtt{L}$

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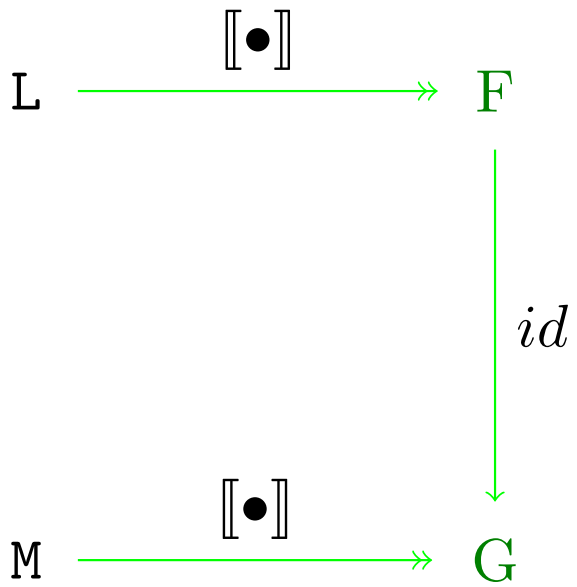

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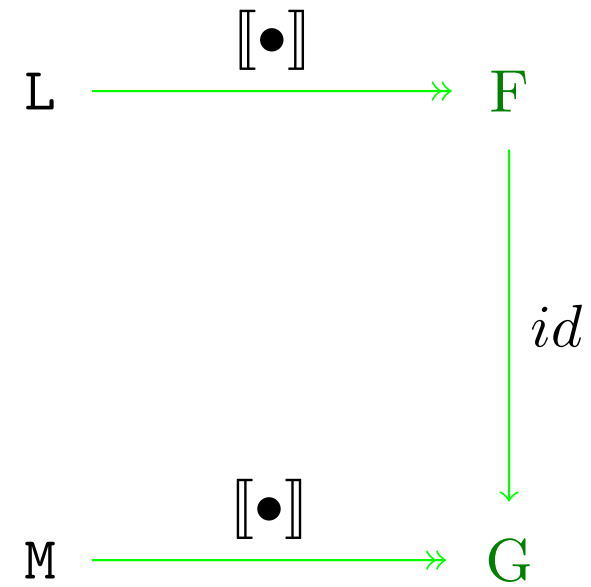

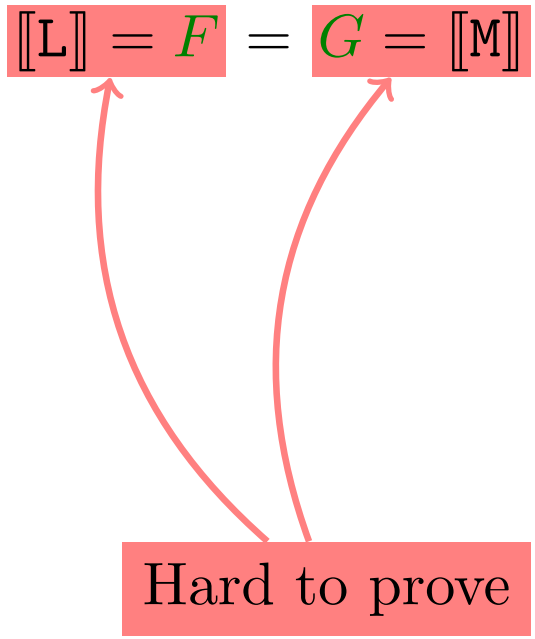

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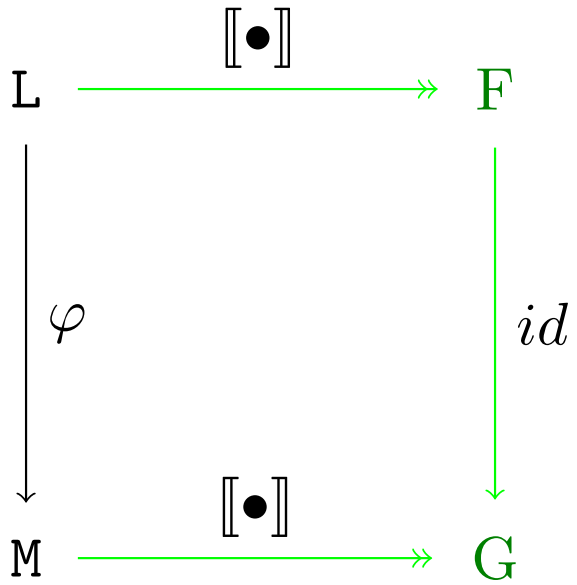
- Compilation $\varphi : \mathtt{L} \to \mathtt{M}$, $[\![\varphi(\mathtt{p})]\!] = [\![\mathtt{p}]\!]$ Semantics preserving



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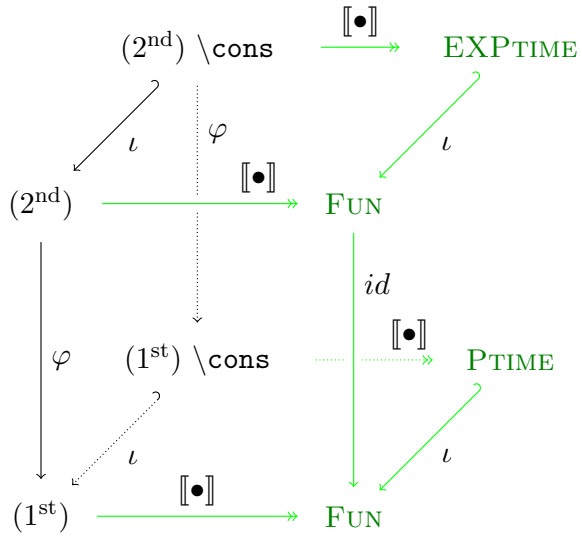
Conclusion

There is no filter-preserving compilation: $\mathtt{p} \in \mathtt{Q} \Leftrightarrow \varphi(\mathtt{p}) \in \mathtt{Q}$

Conversely, if there exists a filter preserving compilation, then $F' = G'$.

High Order



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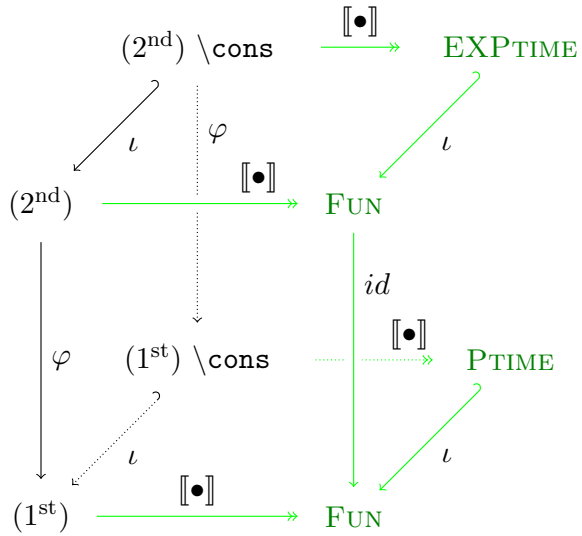

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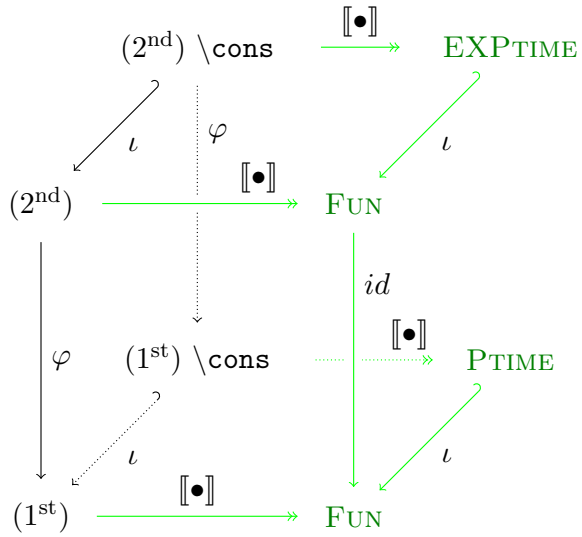

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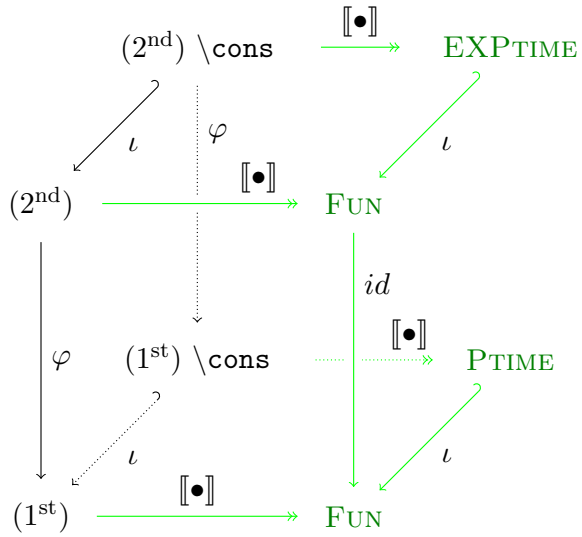

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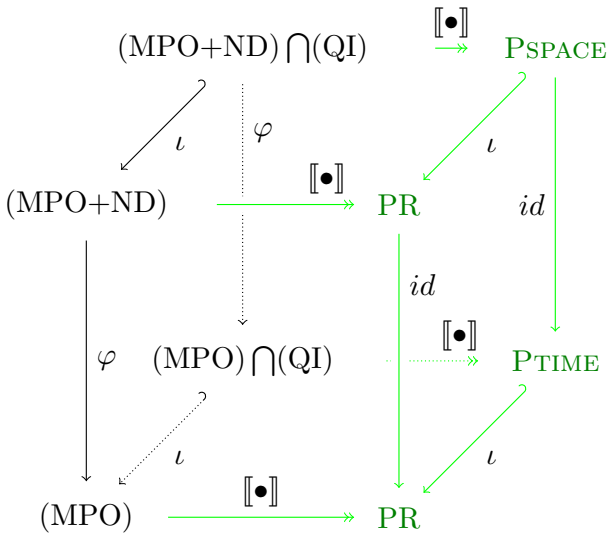

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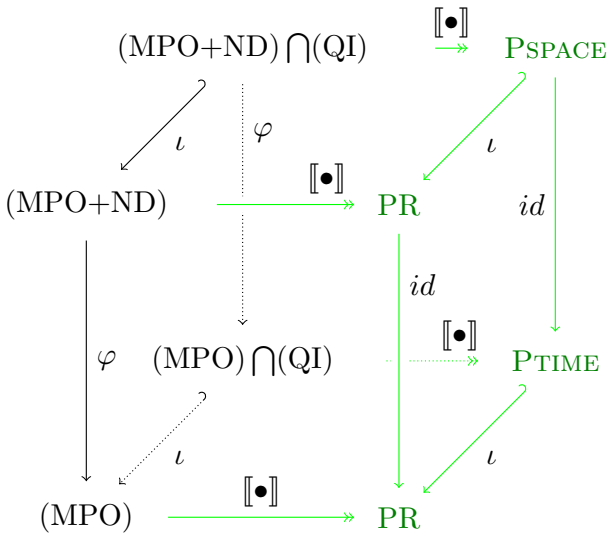

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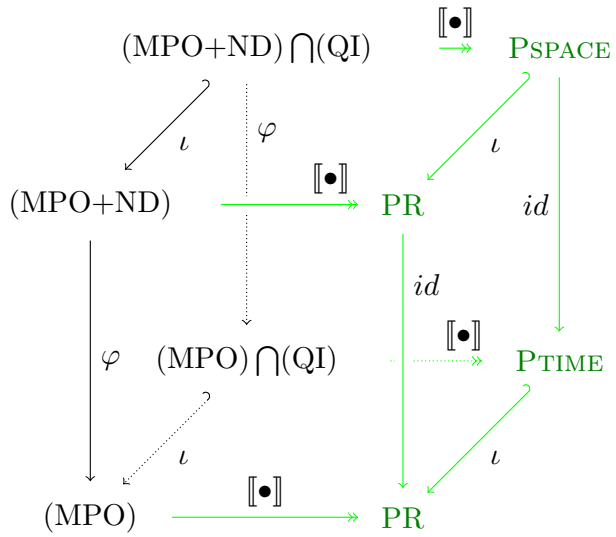

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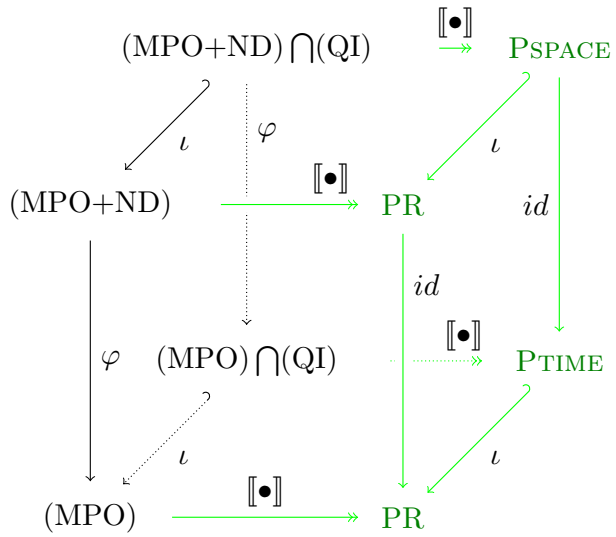

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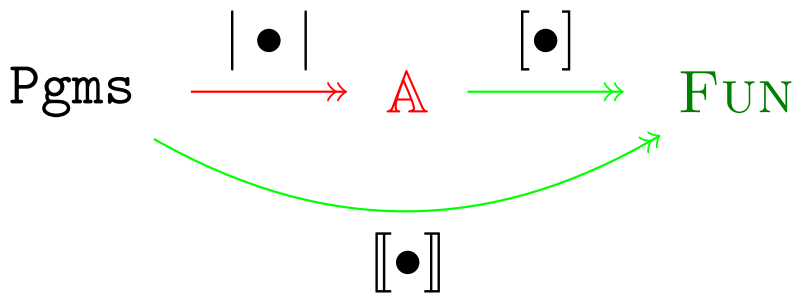
Algorithms lay somewhere between programs and functions.



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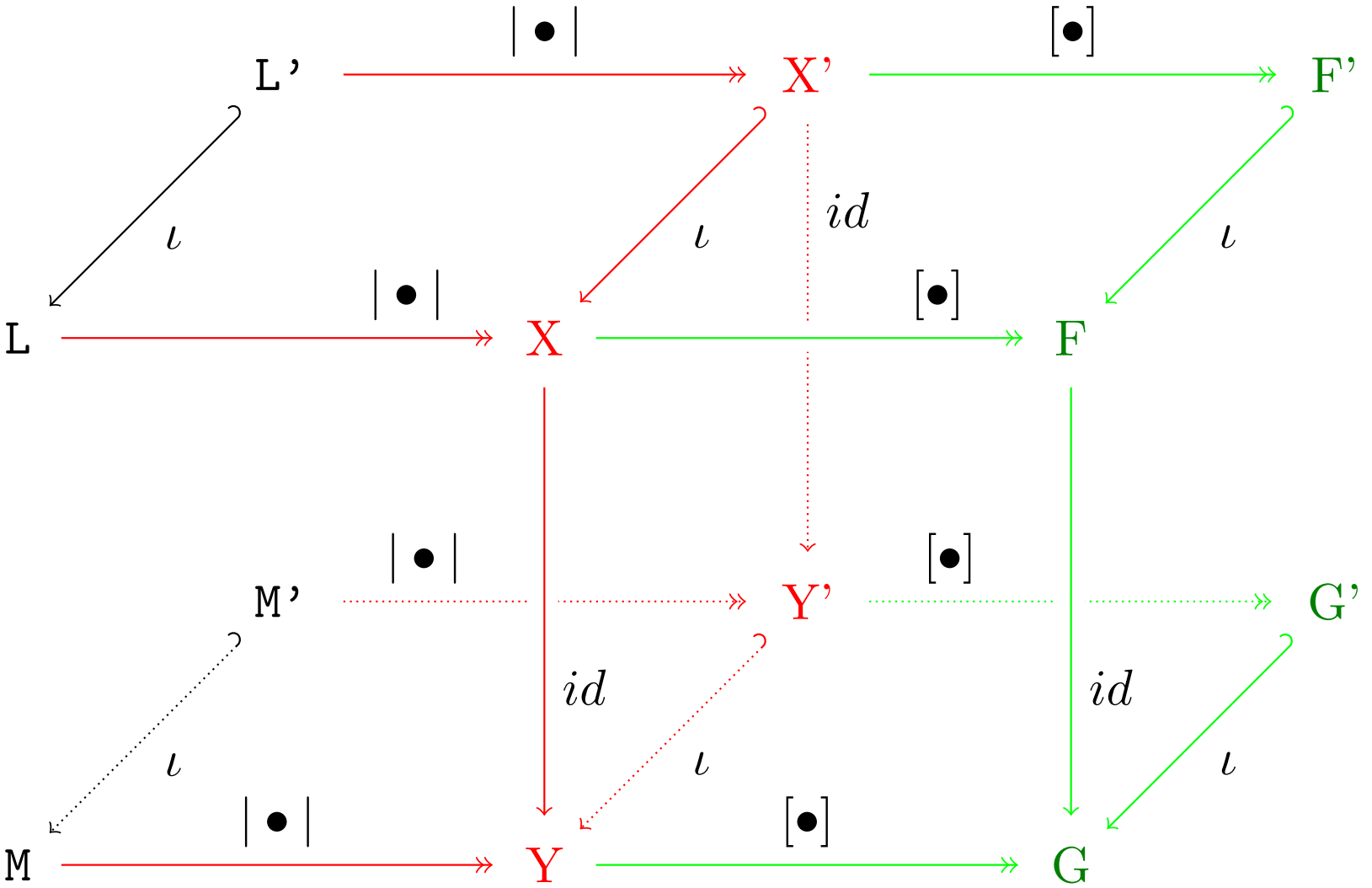
If there exists a filter $\mathtt{Q}$ such that $[\![\mathtt{L} \bigcap Q]\!] = F' \neq G' = [\![\mathtt{M} \bigcap Q]\!]$ then F and G are not algorithmically equivalent.

The big picture



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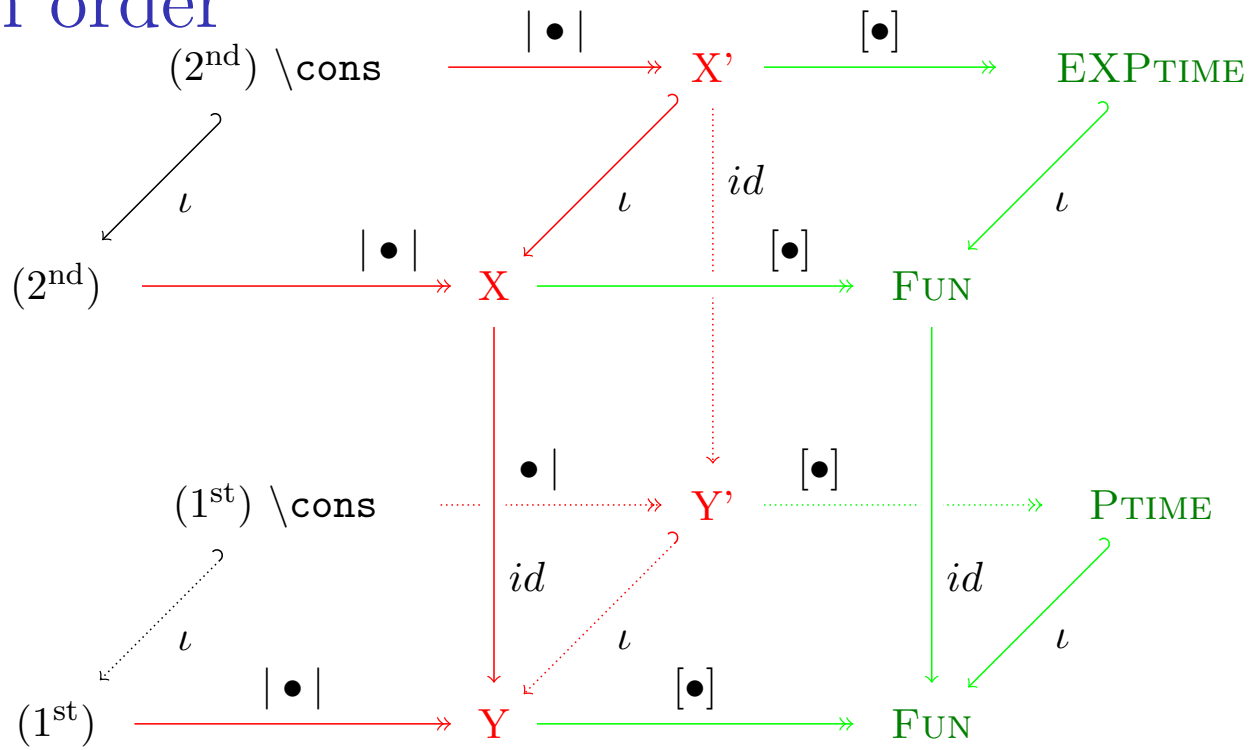

Algorithms respect `cons`-free.

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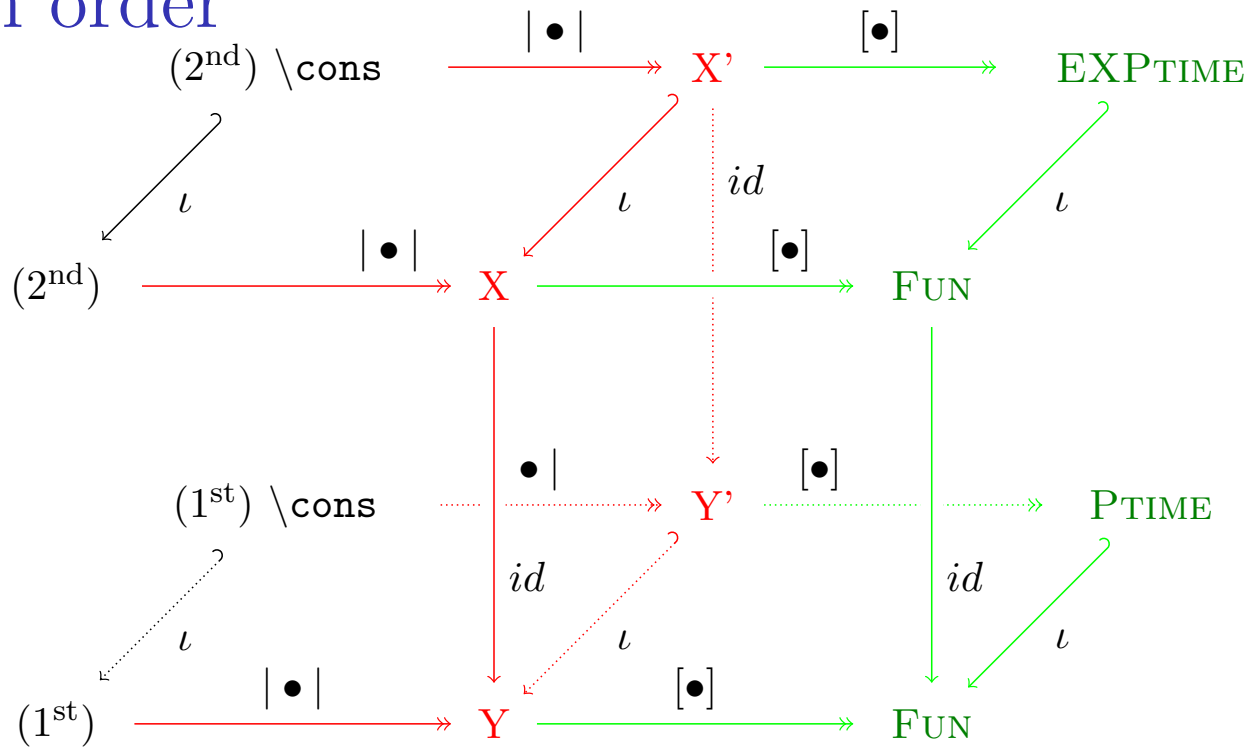

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There are `cons`-free high-order algorithms with no first order counterpart.

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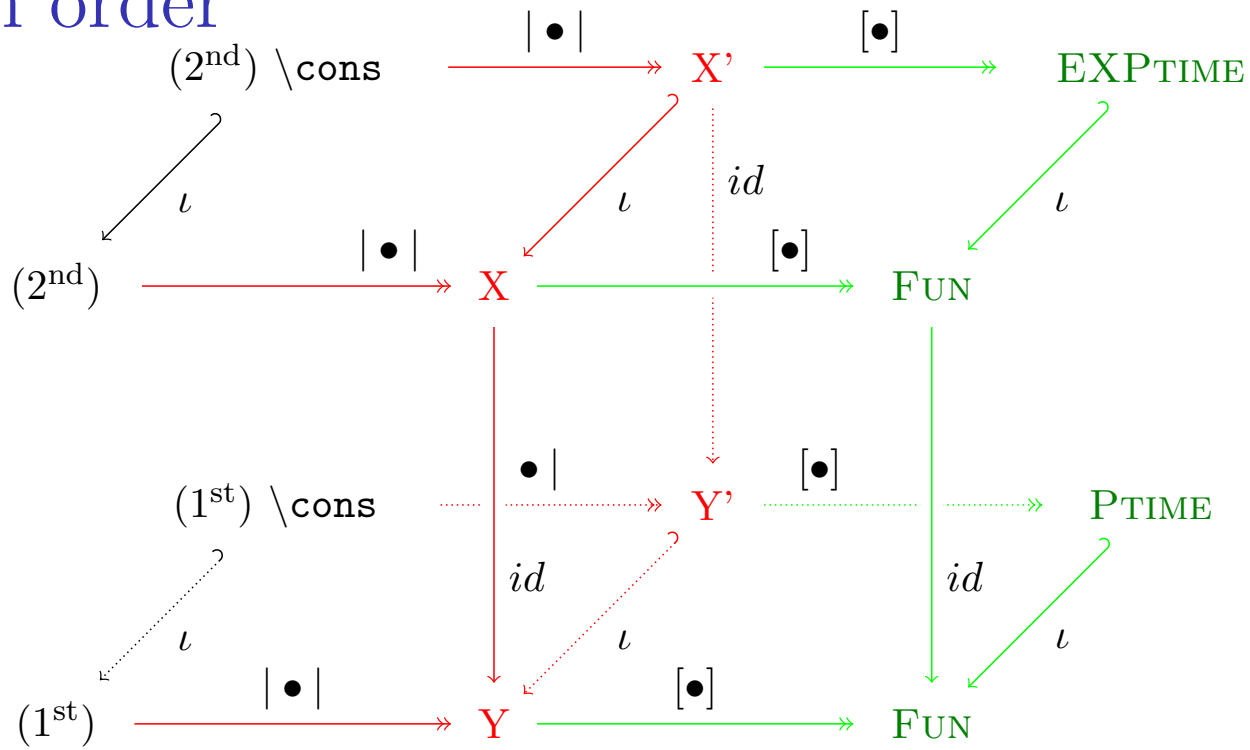

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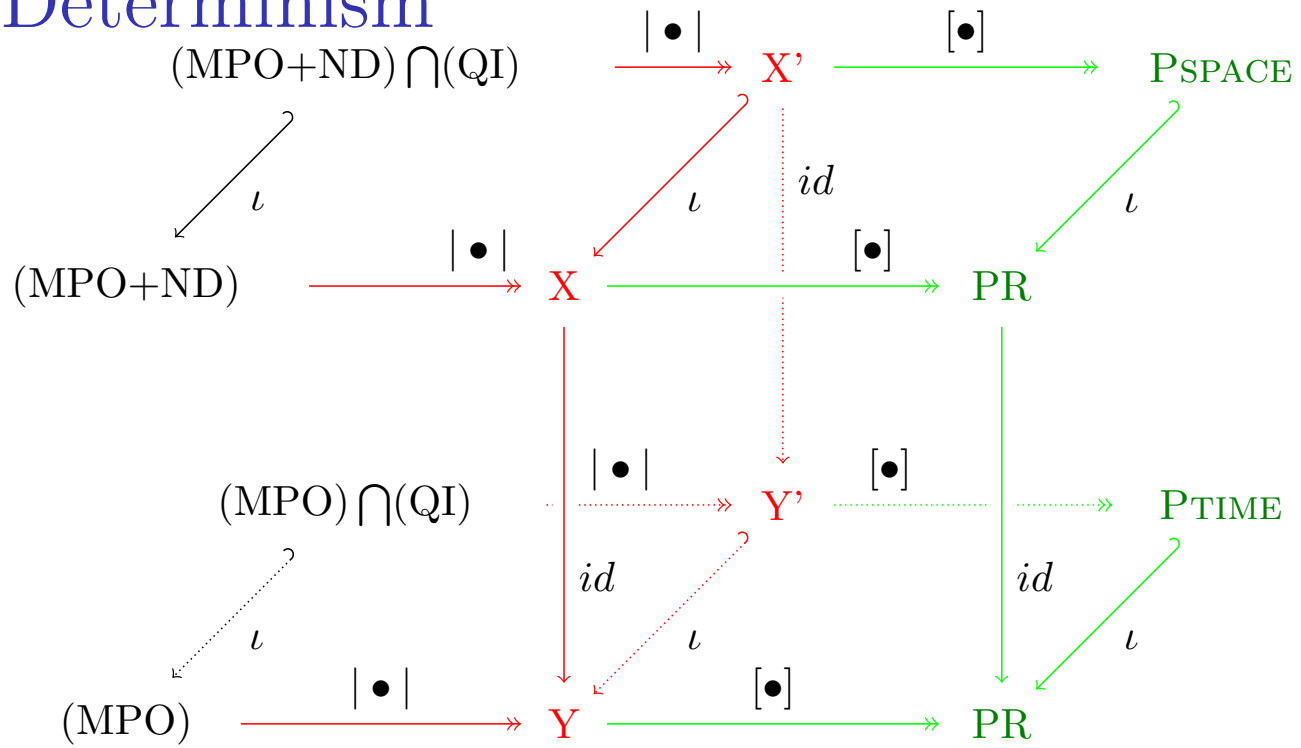

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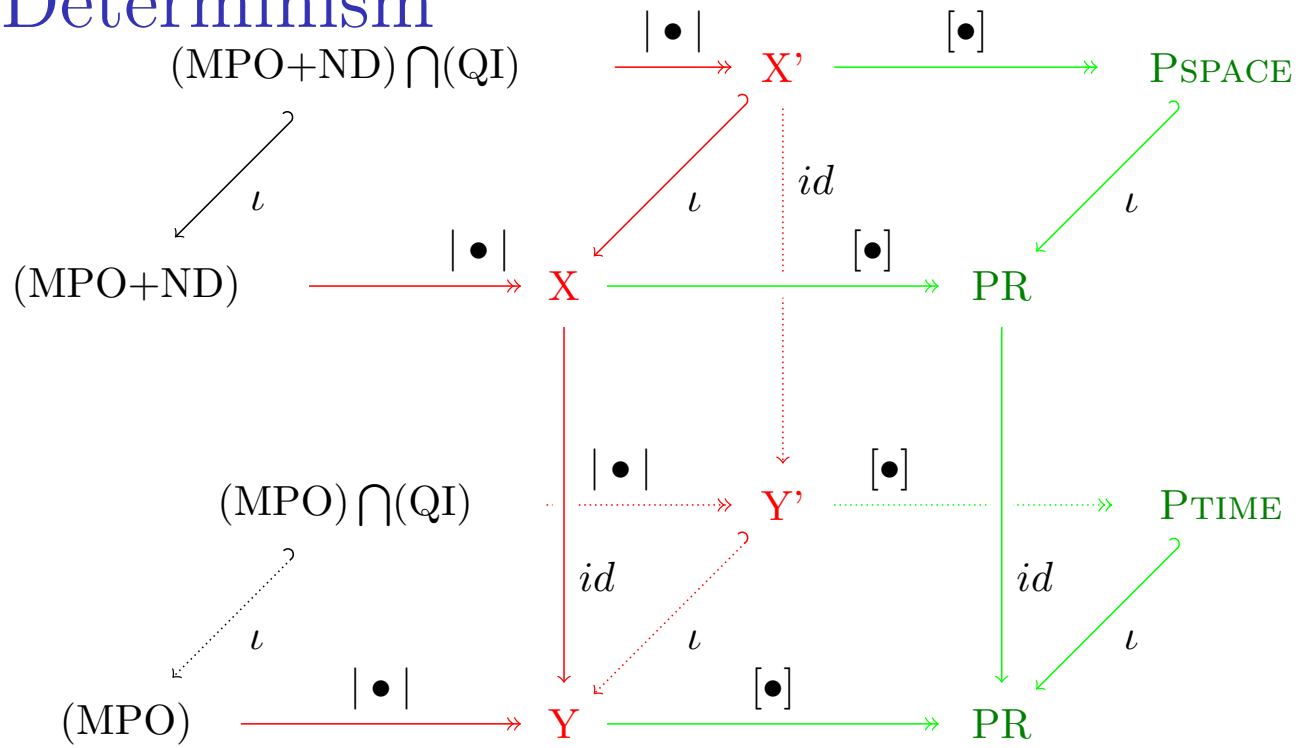

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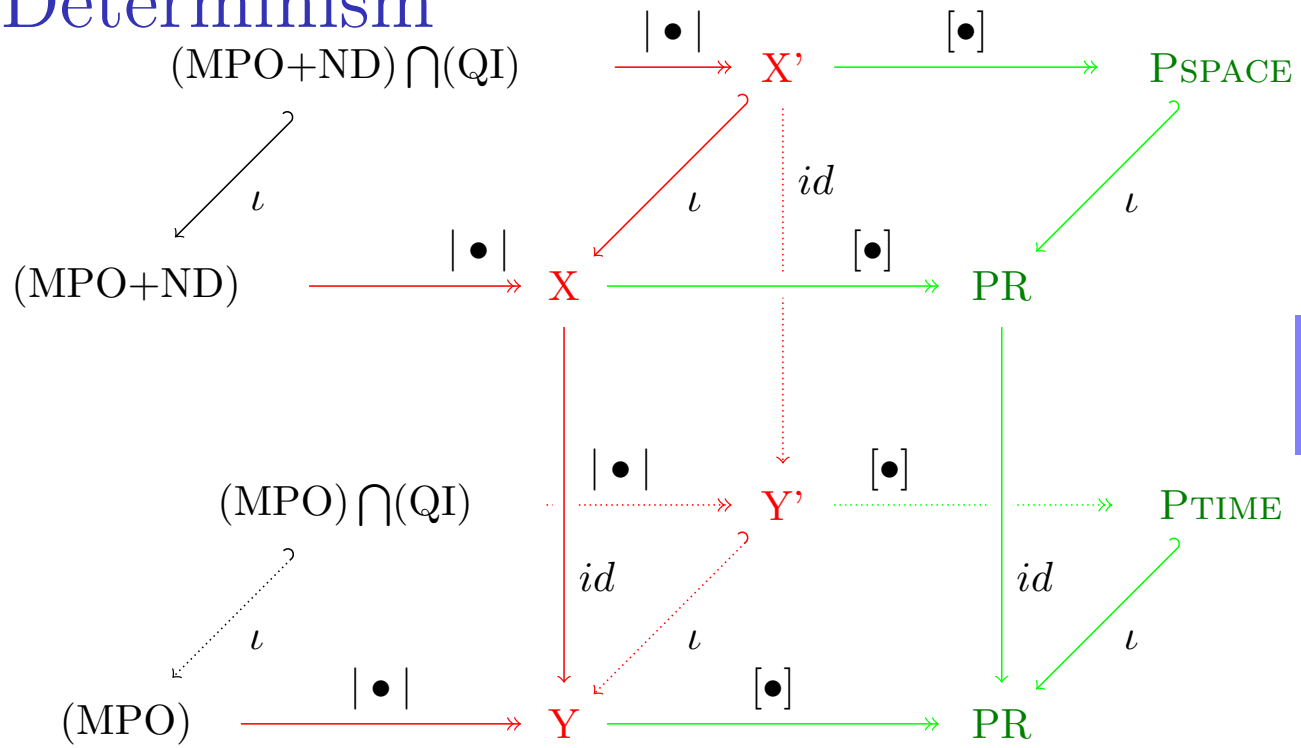

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Chemistry

A chemical version of this:

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- The framework is generic and could be applied to many cases, with little assumptions about "algorithms"
- But we still need to find a good filter and prove:
  $[\![\mathtt{L}]\!] = [\![\mathtt{M}]\!]$
  $[\![\mathtt{L'}]\!] \neq [\![\mathtt{M'}]\!]$
- Reuse 20 years of ICC to get more results