Thick Subtrees, Games and Experiments

Pierre Boudes
LIPN Paris 13, CNRS
TLCA 09, Brasilia

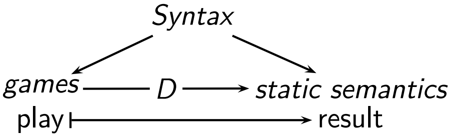
July 2, 2009

Background

- Comparing static and dynamic semantics
  - Hypercoherences form the extensional collapse of sequential algorithms (Berry, Curien, 1982), Ehrhard, 1996.
  - Timeless games, Baillot, Danos, Ehrhard, Regnier, 1997.
- Extensional games, Paul-André Melliès, 2005.
- Hypercoherences unfolding and bordered games, Boudes, 2004.

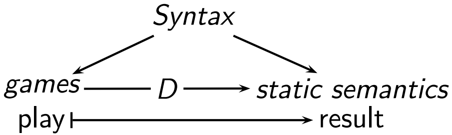
Background

- Comparing static and dynamic semantics
  - Hypercoherences form the extensional collapse of sequential algorithms (Berry, Curien, 1982), Ehrhard, 1996.
  - Timeless games, Baillot, Danos, Ehrhard, Regnier, 1997.
- Extensional games, Paul-André Melliès, 2005.
- Hypercoherences unfolding and bordered games, Boudes, 2004.
- Searching a simple way to project games onto static semantics:



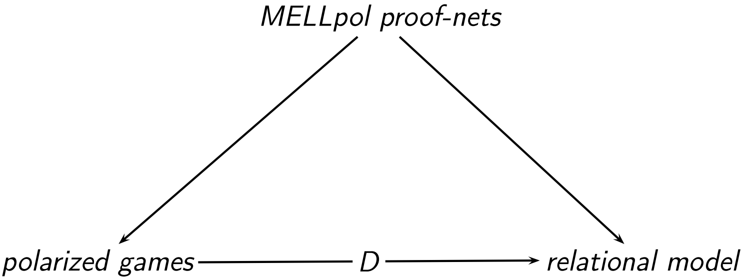
Background

- Comparing static and dynamic semantics
  - Hypercoherences form the extensional collapse of sequential algorithms (Berry, Curien, 1982), Ehrhard, 1996.
  - Timeless games, Baillot, Danos, Ehrhard, Regnier, 1997.
- Extensional games, Paul-André Melliès, 2005.
- Hypercoherences unfolding and bordered games, Boudes, 2004.
- Searching a simple way to project games onto static semantics:

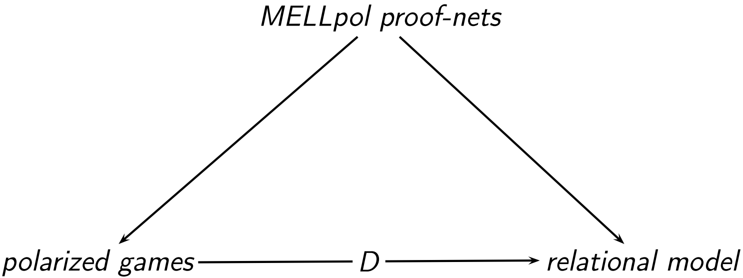


  - Ease of use
  - Pulling/pushing properties of semantics through D
  - Injectivity of the relational model
  - Implicit computational complexity

Statement

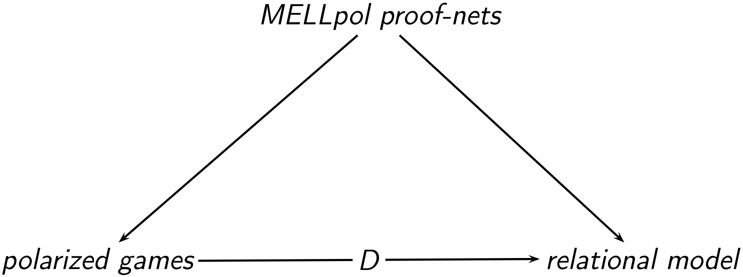


Statement



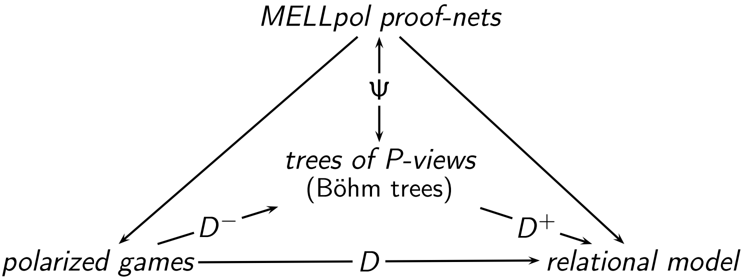
- Polarized games are full and faithful (Laurent, 2002)

Statement



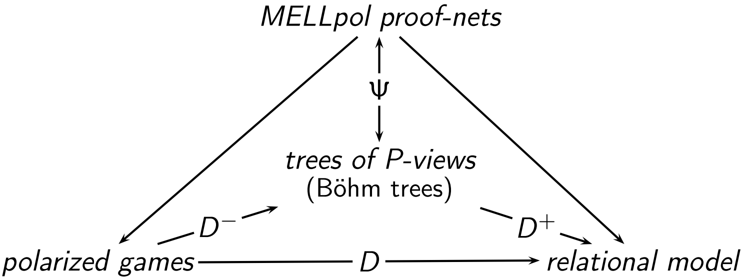
- Polarized games are full and faithful (Laurent, 2002)
- Here MELLpol, but extension to LLpol easy

Statement



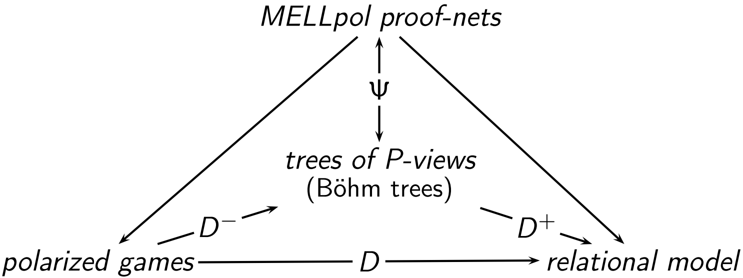
- Polarized games are full and faithful (Laurent, 2002)
- Here MELLpol, but extension to LLpol easy

Statement



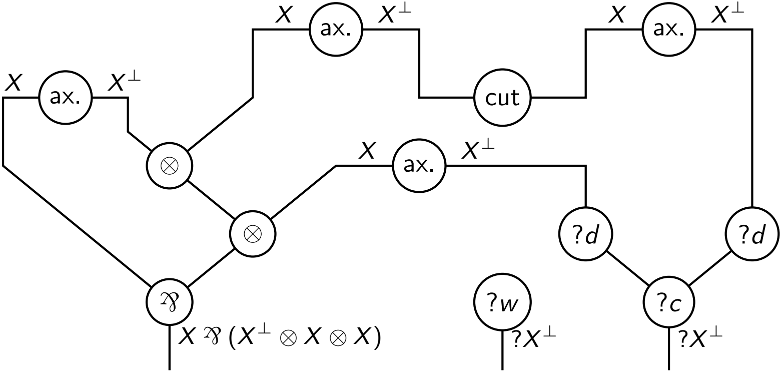
- Polarized games are full and faithful (Laurent, 2002)
- Here MELLpol, but extension to LLpol easy
- Girard's experiments play a central role

Statement



- Polarized games are full and faithful (Laurent, 2002)
- Here MELLpol, but extension to LLpol easy
- Girard's experiments play a central role
- A convenient notion : thick subtrees

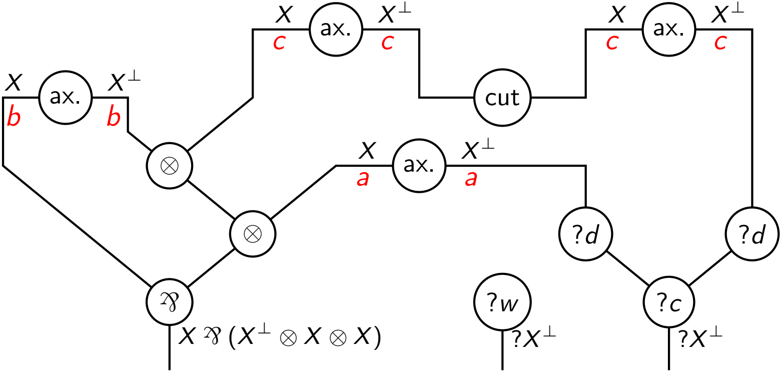
Experiments in linear logic



The interpretation of π is the set of results of experiments on π

Experiments in linear logic

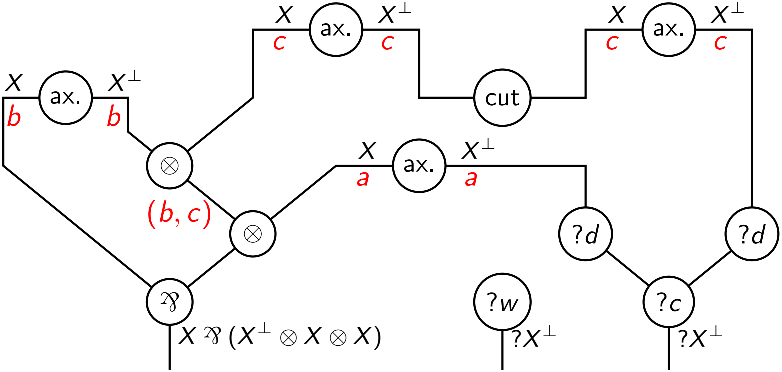
$a, b, c \in |X|$



The interpretation of π is the set of results of experiments on π

Experiments in linear logic

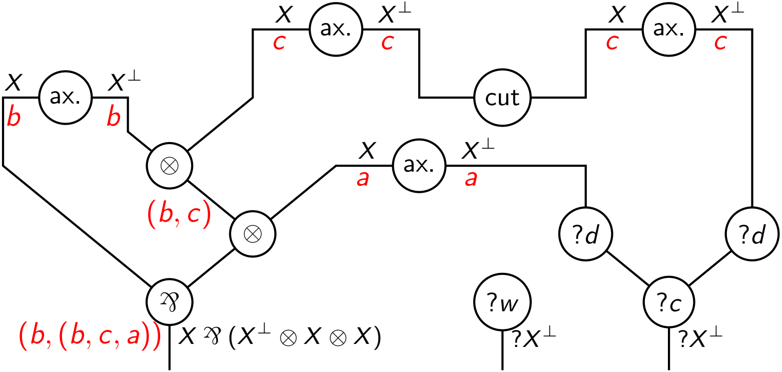
$a, b, c \in |X|$



The interpretation of π is the set of results of experiments on π

Experiments in linear logic

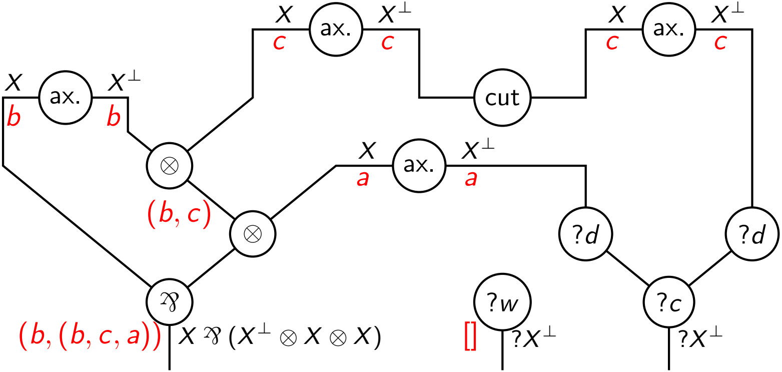
$a, b, c \in |X|$



The interpretation of π is the set of results of experiments on π

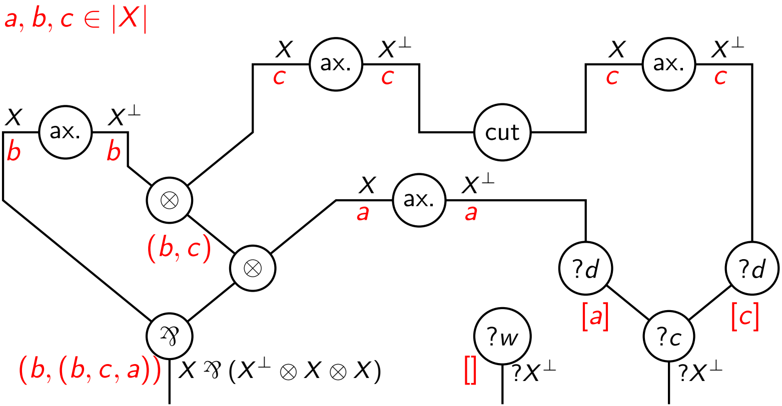
Experiments in linear logic

$a, b, c \in |X|$



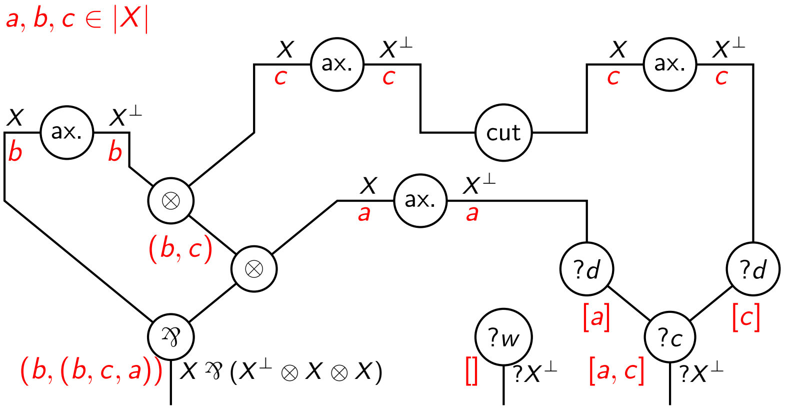
The interpretation of π is the set of results of experiments on π

Experiments in linear logic



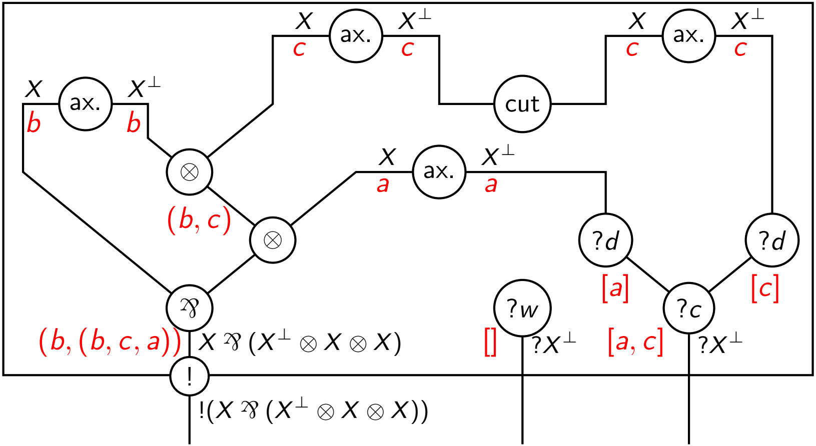
The interpretation of π is the set of results of experiments on π

Experiments in linear logic

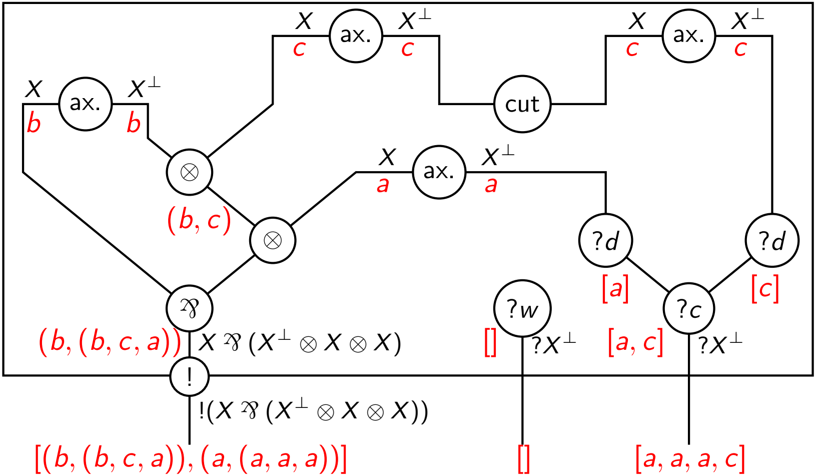


The interpretation of π is the set of results of experiments on π

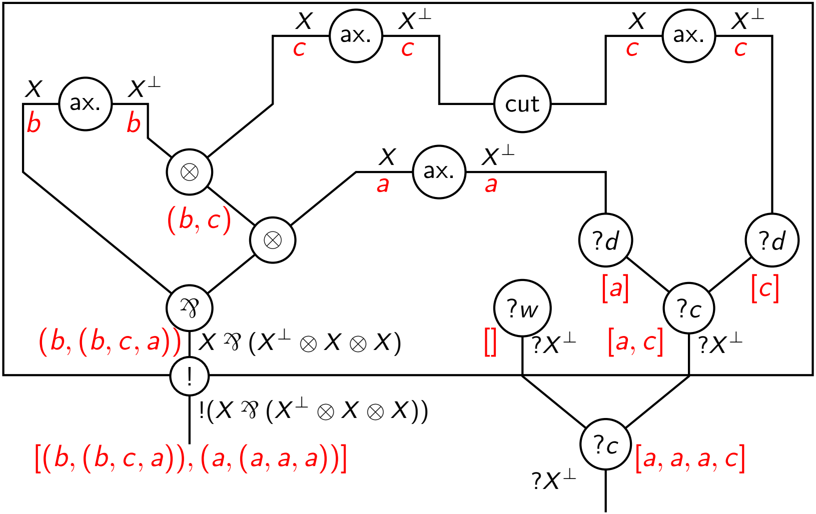
Experiments in linear logic



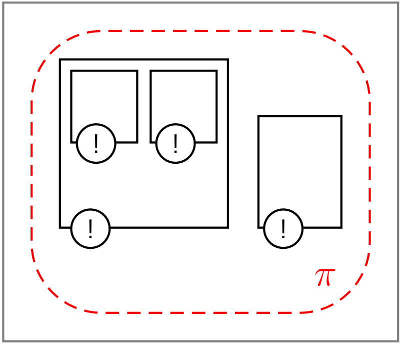
Experiments in linear logic

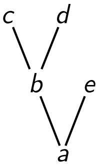


Experiments in linear logic



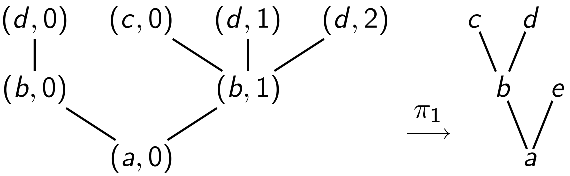
The tree of nested boxes





Thick subtrees

- A tree morphism $f : s \to t$ is a function s.t. :
  - $f(\text{root}(s)) = \text{root}(t)$
  - $a <_s^1 b \implies f(a) <_t^1 f(b)$.
- A thick subtree of t is a tree s together with a tree morphism $f : s \to t$.
- Example :



Experiments with thick subtrees

An experiment on a proof-net π is given by :

- A thick subtree s of the nested boxes tree of π.
- A valuation of axioms of s (*ie.* for every axiom occuring in s, the choice of an element of the set interpreting the atom introduced by the axiom)

If π is cut free every such experiment has a result.

Linear Logic with Polarities

Formulæ :

$$N := ?X^{\perp} \mid \perp \mid N \parr N \mid ?P \qquad \text{(negative formulæ)}$$

$$P := !X \mid 1 \mid P \otimes P \mid !N \qquad \text{(positive formulæ)}$$

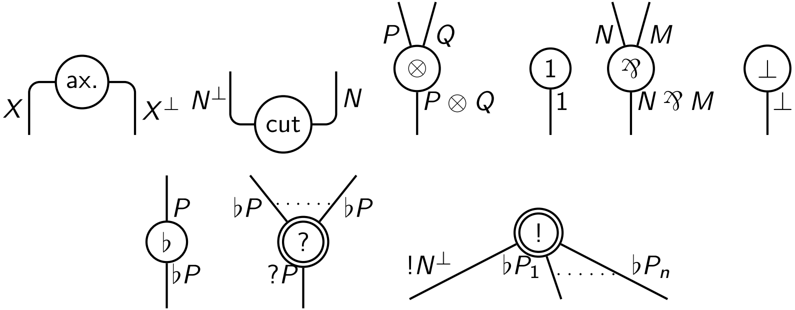
Linear Logic with Polarities

Formulæ :

$$N := ?X^{\perp} \mid \perp \mid N \parr N \mid ?P \qquad \text{(negative formulæ)}$$

$$P := !X \mid 1 \mid P \otimes P \mid !N \qquad \text{(positive formulæ)}$$

Proof-nets :



Anatomy of cut free proof-nets

Let π be a cut free MELLpol proof-net.

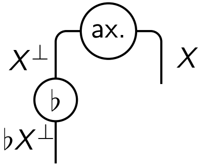
Anatomy of cut free proof-nets

Let π be a cut free MELLpol proof-net. Then each !-box of π contains :

Anatomy of cut free proof-nets

Let π be a cut free MELLpol proof-net. Then each !-box of π contains :

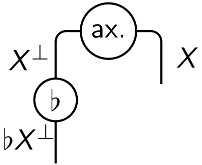
- either an axiom followed by a $\flat$-link on its positive conclusion

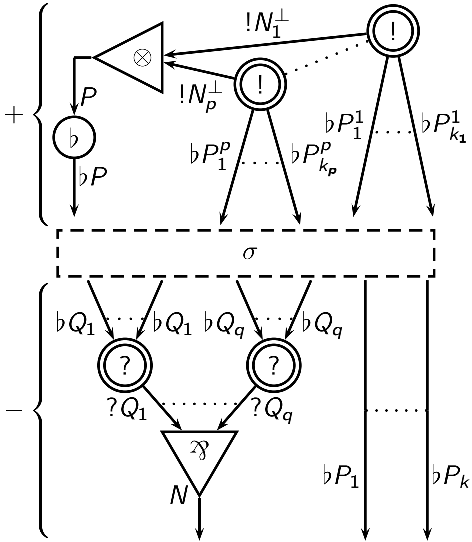


Anatomy of cut free proof-nets

Let π be a cut free MELLpol proof-net. Then each !-box of π contains :

- either an axiom followed by a $\flat$-link on its positive conclusion
- or a net as beside

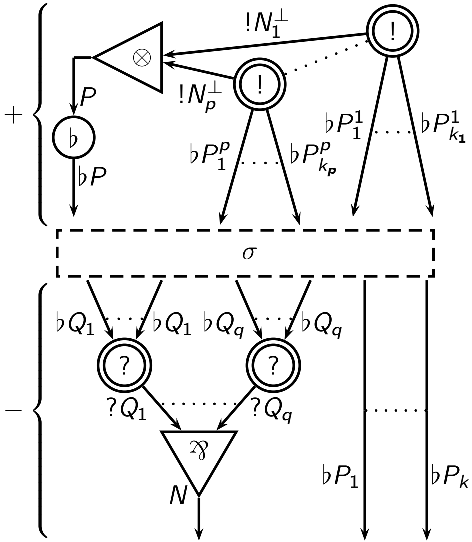




Anatomy of cut free proof-nets

Let π be a cut free MELLpol proof-net. Then each !-box of π contains :

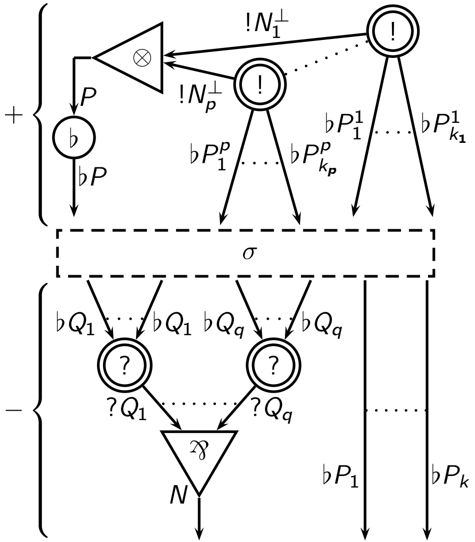
- either an axiom followed by a $\flat$-link on its positive conclusion
- or a net as beside
- in both cases there are only one (big) positive connective, one (big) negative connective and wiring



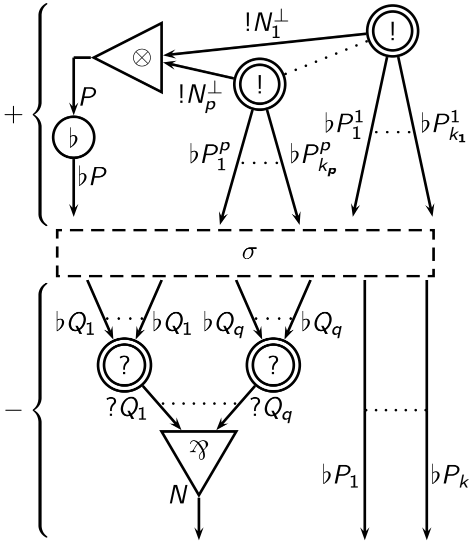
Anatomy of cut free proof-nets

Let π be a cut free MELLpol proof-net. Then each !-box of π contains :

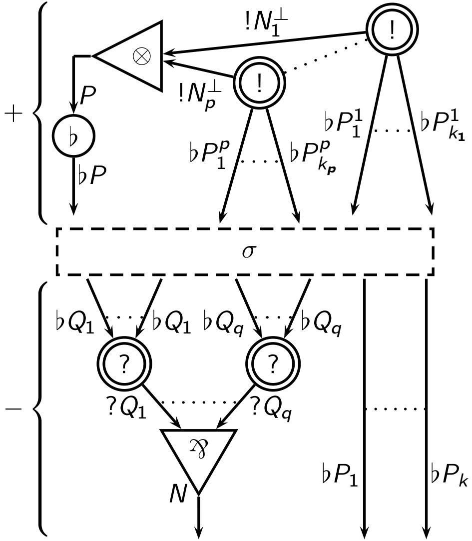
- either an axiom followed by a $\flat$-link on its positive conclusion
- or a net as beside
- in both cases there are only one (big) positive connective, one (big) negative connective and wiring
- if π has only one negative conclusion, the same holds at depth zero.

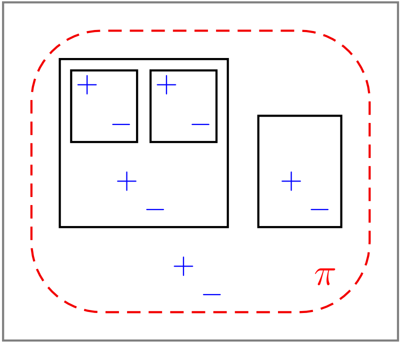


Connecting things

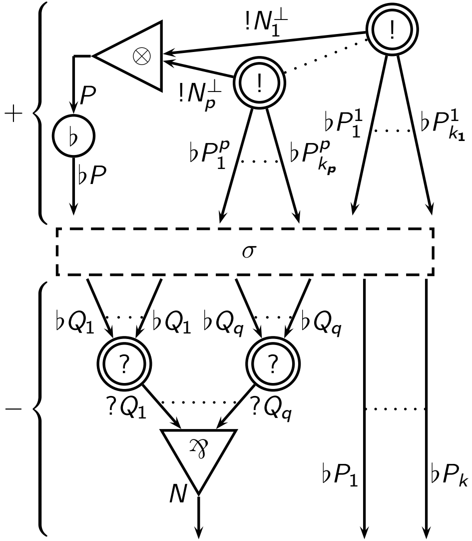


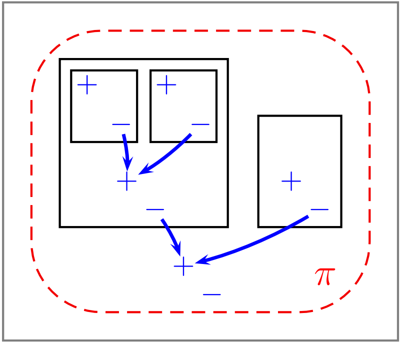
Connecting things



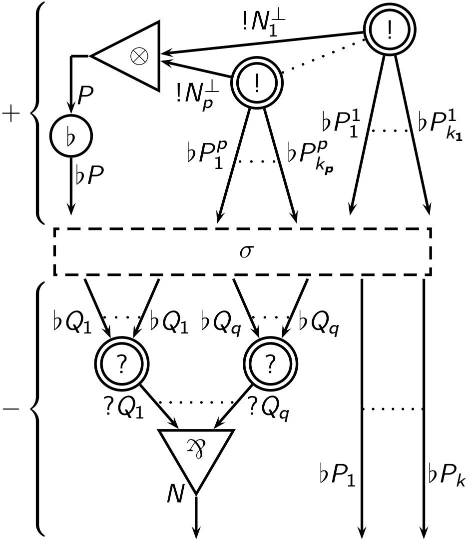


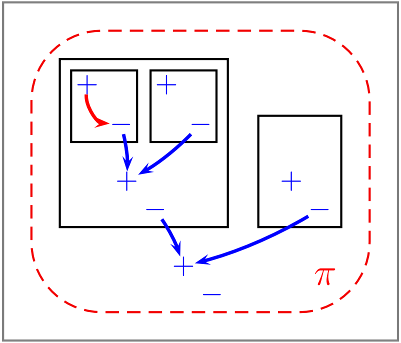
Connecting things



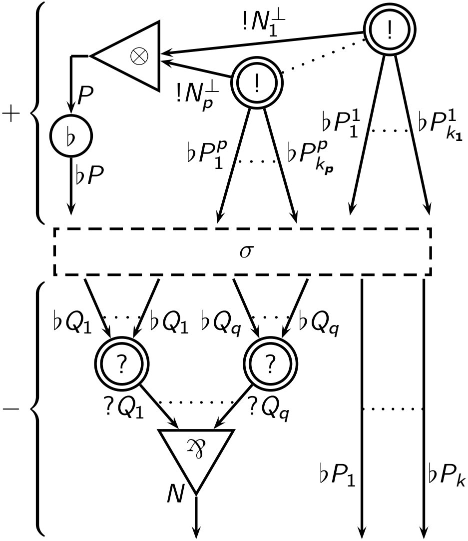


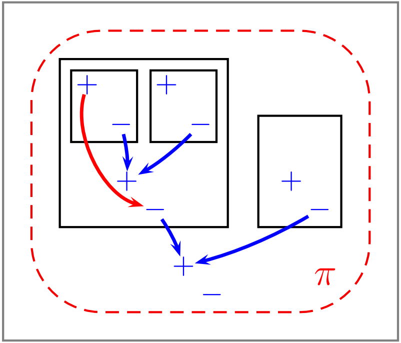
Connecting things



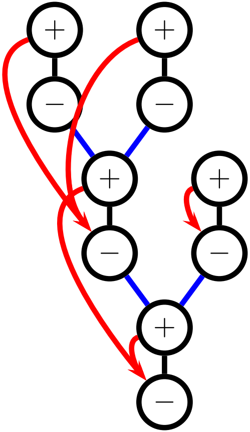


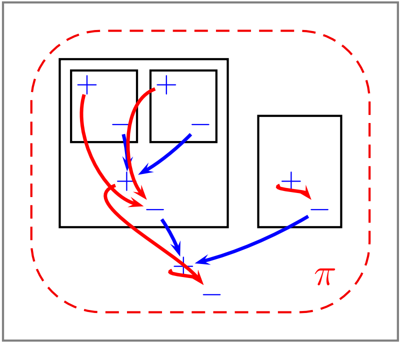
Connecting things





Connecting things





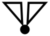
Games semantics: Arenas

Arena of a formula:

$\text{arena}(X) = \bullet(X)$

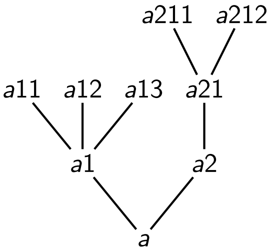
$\text{arena}(1) = \bullet$

$\text{arena}(P) =$  $\Longrightarrow \text{arena}(?P) =$ 

$\begin{cases} \text{arena}(P) = \end{cases}$ 
$\text{arena}(P') =$  $\Longrightarrow \text{arena}(P \otimes P') =$ 

$\text{arena}(P^{\perp}) = \text{arena}(P)$

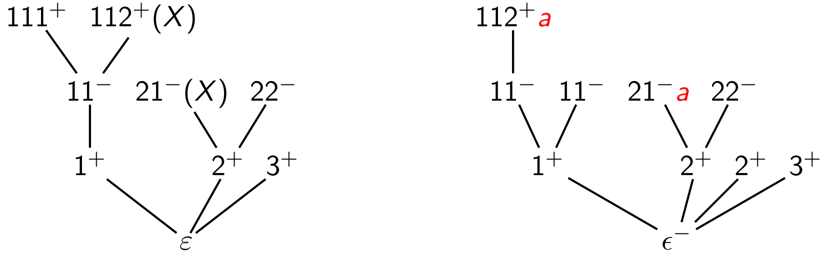
Naming:



a word of integer

Arenas and results of experiments

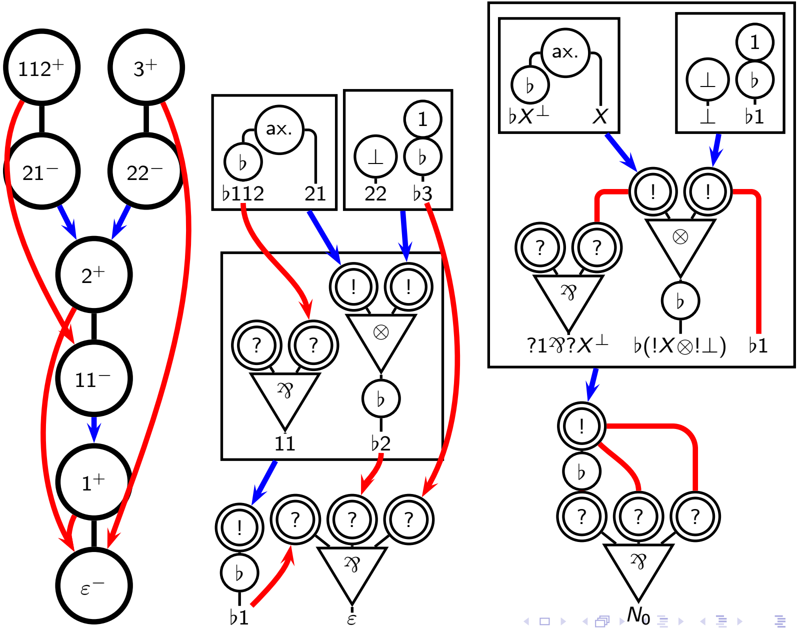
$$N_0 = ?!(?1 \parr ?X^{\perp}) \parr ?(!X \otimes !\perp) \parr ?1$$



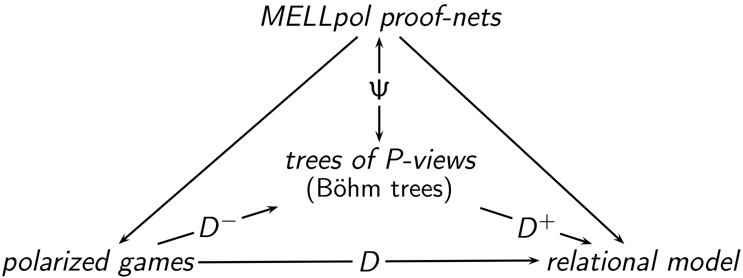
$$r = ([\ [([\ ],[a])],\ [\ ]\ ],\quad [([a],[*]),([\ ],[\ ])],\quad [*])$$

Up to the valuation of atoms there is a correspondence between thick subtrees of an arena and results of the corresponding type.

Ψ: from trees of P-views to PNs and back



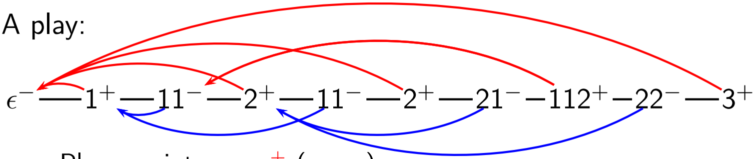
Statement



- Polarized games are full and faithful (Laurent, 2002)
- Here MELLpol, but extension to LLpol easy
- Girard's experiments play a central role
- A convenient notion : thick subtrees

Visibility, views and plays

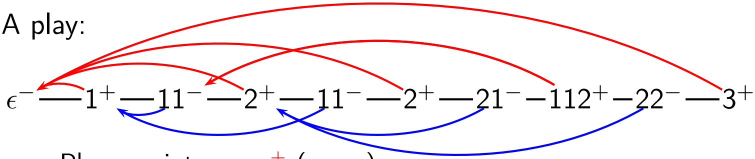
- A play:



  - Player pointers: $\leftarrow^+$ (arena)
  - Opponent pointers: $\leftarrow^-$ (arena)
  - Player precedence: $<_1^+$ (being in the same box)
  - Opponent precedence : $<_1^-$

Visibility, views and plays

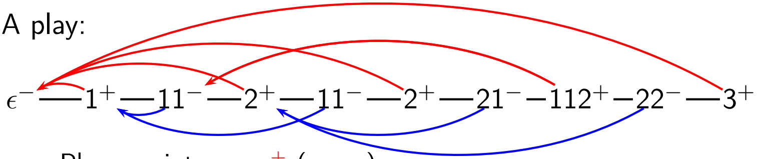
- A play:



  - Player pointers: $\leftarrow^+$ (arena)
  - Opponent pointers: $\leftarrow^-$ (arena)
  - Player precedence: $<_1^+$ (being in the same box)
  - Opponent precedence : $<_1^-$
- Visibility condition: $\leftarrow^+ \subset (<_1^+ \cup \leftarrow^-)^*$ *(Dually covisibility).*
- The P-view $D^-(p)$ of a visible play p is the tree $(<_1^+ \cup \leftarrow^-)^*$ together with the relation $\leftarrow^+$ *(Dually D^+).*

Visibility, views and plays

- A play:

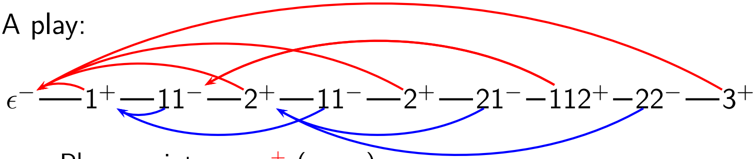


  - Player pointers: $\leftarrow^{+}$ (arena)
  - Opponent pointers: $\leftarrow^{-}$ (arena)
  - Player precedence: $<_1^{+}$ (being in the same box)
  - Opponent precedence : $<_1^{-}$

- Visibility condition: $\leftarrow^{+} \subset (<_1^{+} \cup \leftarrow^{-})^*$ *(Dually covisibility).*
- The P-view $D^{-}(p)$ of a visible play p is the tree $(<_1^{+} \cup \leftarrow^{-})^*$ together with the relation $\leftarrow^{+}$ *(Dually D^{+}).*
- Let ϕ be a innocent strategy, given by its tree of P-views. When $D^{-}(p)$ is a TST of ϕ, p is a play of ϕ.

Visibility, views and plays

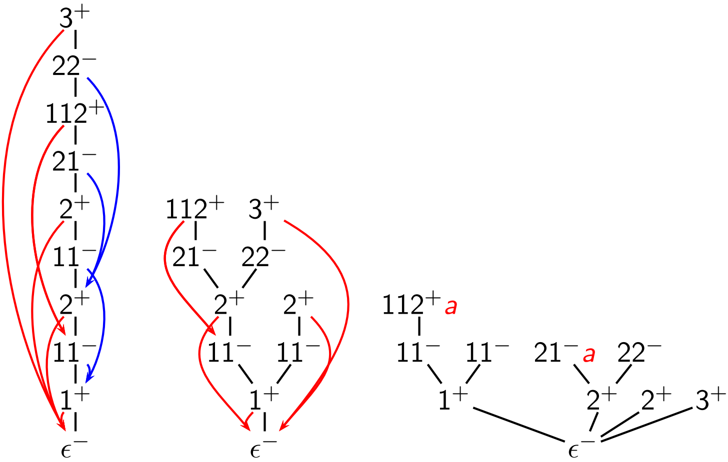
- A play:



  - Player pointers: $\leftarrow^{+}$ (arena)
  - Opponent pointers: $\leftarrow^{-}$ (arena)
  - Player precedence: $<_1^{+}$ (being in the same box)
  - Opponent precedence : $<_1^{-}$

- Visibility condition: $\leftarrow^{+} \subset (<_1^{+} \cup \leftarrow^{-})^{*}$ *(Dually covisibility).*
- The P-view $D^{-}(p)$ of a visible play p is the tree $(<_1^{+} \cup \leftarrow^{-})^{*}$ together with the relation $\leftarrow^{+}$ *(Dually D^{+}).*
- Let ϕ be a innocent strategy, given by its tree of P-views. When $D^{-}(p)$ is a TST of ϕ, p is a play of ϕ.
- The full desenquentialization $D(p)$ of p is the tree $(\leftarrow^{+} \cup \leftarrow^{-})^{*}$. It is a TST of the arena.

Play, experiment, result



Conclusion

- Polarized games are just a more compact syntax for cut-free proofs and we have a really direct encoding in both directions
- This cannot be extended easily to linear logic without polarities
- Direct reading of properties of semantics through D (equitability, neutrality)
- Further works:
  - Injectivity of the relational model for LLpol
  - With Paolo di Giamberardino, a semantics for a syntax with boxes overlap (thick subdags?)

Thank you